DIVISION OF
FLUID, THERMAL AND AEROSPACE SCIENCES

SCHOOL OF ENGINEERING

CASE WESTERN RESERVE UNIVERSITY

UNIVERSITY CIRCLE • CLEVELAND, OHIO 44106

FTAS/TR-68-33

MEASUREMENT OF TURBULENCE QUANTITIES BY PRESSURE PROBES

by

Yasuhiro Kamotani and H. Kenneth Wiskind

CASE WESTERN RESERVE UNIVERSITY

September 1968

ABSTRACT

As an instrument to measure turbulence intensities and shear stress, the pressure probe was developed by Jezdinsky[1]. His original equation, on which the pressure probe is based, is erroneous, and it is derived here in a more rigorous manner. It is found that the value of turbulent shear stress computed from Jezdinsky's equation is too small.

The pressure probe method necessitates measuring the mean velocity and the static pressure by other instruments. In the present discussion the mean total and static pressures in a turbulent flow are measured by a Pitot tube.

The pressure probe is tested by measuring turbulence quantities in a round free jet, and the results are compared with those taken by a hot-wire anemometer. The agreement is satisfactory.

ACKNOWLEDGEMENTS

The authors wish to thank the National Aeronautics and Space Administration for providing financial support under Grant number NGR - 36 - 003 - 064.

TABLE OF CONTENTS

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

D	=	diameter of nozzle mouth
$\bar{P}$, $\bar{P}_s$	=	mean value of pressure
$\bar{P}_{sm}$, $\bar{P}_{tm}$	=	measured mean value of pressure
$\bar{U}$	=	axial component of mean velocity
$\bar{V}$	=	radial component of mean velocity
U_R	=	$\sqrt{\bar{U}^2 + \bar{V}^2}$
U_{eff}	=	total velocity in a turbulent flow
U_o	=	velocity at the mouth
$\bar{U}_m$	=	velocity on the axis of a jet
p, p_s, p_{sm}	=	turbulent fluctuation of pressure
Δp_w	=	pressure drop by the third velocity component w
r	=	radial distance from jet axis
$r_{1/2}$	=	the value r at which $\bar{U} = \frac{1}{2}\bar{U}_m$
u,v,w	=	turbulent velocity
u′	=	$\sqrt{\bar{u}^2}$
v′	=	$\sqrt{\bar{v}^2}$
x	=	axial distance from nozzle
α	=	angle between the axis of the probe and the mean flow direction
α′	=	turbulent fluctuation of the angle of attack
β	=	steady yaw angle in the lateral direction
θ	=	angle between the steady flow and the pressure probe

INTRODUCTION

CHAPTER I.

The hot-wire anemometer is a well-developed and the most reliable instrument for measuring turbulence. But in the turbulent flows of high intensity a hot-wire measurement is no longer correct because of the deviation from the linear heat transfer equation on which a hot-wire anemometer is based.

To measure turbulence of high intensity simply but with a reasonable accuracy the pressure probe has been developed by Jezdinsky [1]. The essential principle of this instrument is the following. A pair of fine tubes with beveled ends, placed back to back is often used as a wind direction meter. The flow direction is determined by rotating these tubes in their axial plane until the pressures in the mouths of two tubes are equal, this condition being indicated on a differential manometer to which they are connected. The direction angle is determined with an accuracy of about 0.1 degree when the flow is nonturbulent or its turbulence level is very low. If the flow is turbulent, this wind direction meter yields erroneous results. Since this error depends on turbulence intensity and turbulent shear stress, we can use such instrument as a means of turbulence measurement. The theory and the structure of pressure probes are discussed in Chapter II.

Some advantages of pressure probes compared to a hot-wire anemometer are:

1. The structure of pressure probes is very simple, and the electric circuit which gives some effects on turbulence measurements is unnecessary.
2. The hot-wire anemometer is very fragile, and special care is necessary to avoid breaking the wire. Since the pressure probe has no such sensitive part, handling is easy, and its life is long.
3. The application of the hot-wire is practically limited to gases. Some problems arise in using it in liquids. In contrast to the hot-wire the pressure probe can be used in any liquid without difficulty.
4. The hot-wire is sensitive to accumulation of small particles (dust). Thus the wire must be cleaned and recalibrated. The tip of the pressure probe is slightly sensitive to dust, but its cleaning is very simple.

Besides the measurements by pressure probes it is necessary to measure mean values of both total and static pressures at the same point in the flow. The measurements of both pressures are affected by turbulence so that accurate measurement is not a simple matter. This is discussed in Chapter III.

The pressure probe was tested by measuring turbulence quantities of a round free jet, and the results were compared with those taken by a hot-wire anemometer. They are presented in Chapter IV.

CHAPTER II. THE DESIGN AND THE THEORY OF PRESSURE PROBES

A. The Design of Pressure Probes

There are some requirements which must be satisfied by the pressure probe in designing the instrument.

1. The principle of the pressure probe method is to obtain the relationship between the angle of attack and the mean pressure readings. Consequently the pressure probe must be made so that its angle of attack is adjustable and the angle can be measured up to 0.1 degree.

   It is critical that the location of the tip of the probe must stay at one point even if the angle of attack is changed.

2. The angle of bevel at the tip is very important, and after many experiments Jezdinsky found out that the best results are obtained when the angle is about 55 degrees. The shape of two probes must be symmetrical. Actually, however, there will always be small difference between them which produces small error in measurements.

3. Aerodynamically each probe must have the smallest possible diameter and the greatest permissible length, so that only the minimum admissible disturbances of the flow are created. A commercially available tube which satisfies these conditions to some extent is a hypodermic needle whose minimum diameter is 0.028 inches.

4. In order that the instantaneous velocity distribution be uniform in the region occupied by the instrument the size of the tube must be smaller than the micro scale of turbulence. Then the diameter of the tube need not be small if the scale of turbulence is large. If possible it is convenient to use various sizes of tubes depending on the scale of turbulence since the remaining parts of the instrument are the same for any tube.
5. The instrument must be sufficiently strong and rigid so that vibrations caused by the turbulent flow can be avoided.
6. Since very accurate measurements of pressure are necessary in this method, care must be taken to avoid leakage throughout the passage from the tip of the probe to a manometer. If the probes are interchangeable, two or three pressure connections are necessary, and checks for leaks must be made after construction.

In Figure 1 the structure of pressure probes is shown. This instrument consists of two probes, a pressure connection, an angle setting wheel, a vernier and a holder. The probes are made of two fine hypodermic needles with their open ends cut obliquely so that they are sensitive not only to the velocity fluctuation in the logitudinal direction but also to the direction normal to the probe axis. Hypodermic needles used as pressure probes are very slender and long so that they are not sufficiently strong and rigid. Thus, except near the tips, they are supported by a thin steel bar to avoid fluctuations in turbulence. The location of the tip must coincide with the axis of rotation so that it remains unchanged if the probe is rotated to change the angle of attack. The shape of the probe depends upon the space for measuring. The shape which is shown in the figure is convenient when enough space is available, but when the space is limited, for example, when the instrument is used for measuring turbulence in a tube, the shape of the tube should be changed to a convenient one. Pressure connections are made from Cannon Plug type K-22C and Receptacle type K-31S.

Detail is shown in Figure 2. Originally this plug was an electric connection and it was necessary to drill a hole through the pin contact of the plug so that pressure can be transmitted from the plug to the receptacle. In order to avoid leakage from this connection the junction is sealed by rubber and cement. Careful inspections for leaks were made whenever the probes were changed. The receptacle is attached to an angle-setting wheel which can be rotated by hand. The side of the wheel is calibrated in degrees and if it is used with vernier the angle of attack can be adjusted within 0.1 degree. A lock screw is used to secure the angle setting after adjustment. A holder is so made that it permits smooth rotation of the angle setting wheel, and it can be attached to a gage positioner which carries the instrument to any desired position in a turbulent flow. Pressure readings are made through the hypodermic needle and are transmitted to a thicker tube connected to a differential micromanometer.

B. The theory of Pressure Probes.

The basic equations of the pressure probe method can be obtained by making use of the sensitivity characteristics of the pressure probe to deviations of the flow direction. If the pressure probe is put into a nonturbulent flow, it is found that the relationships between the sine of the angle of attack and the pressure readings are approximated for not too large angle of attack by (see Figure 3)

$$\frac{P_1 + P_2}{\frac{1}{2}\rho U^2} = A + B \sin\theta + C \sin^2\theta \qquad (1)$$

$$(\theta < 35^\circ)$$

$$\frac{P_1 - P_2}{\frac{1}{2}\rho U^2} = D \sin\theta \qquad (2)$$

Actual values of constants A,B,C,D depend on the size and the shape of the probe. and must be determined by experiments.

Consider now a turbulent flow with turbulent velocity components u,v, and w, so that the instantaneous value of the total velocity is given by

$$U_{eff} = \sqrt{(\bar{U} + u)^2 + v^2 + w^2} \qquad (3)$$

Because of its shape, the probe is assumed to be sensitive to velocity fluctuations u and v in a longitudinal plane and relatively insensitive to turbulent velocity w in the direction normal to it. Consequently the problem can be treated two-dimensionally, and instead of equation (3) we write (see Figure 4).

$$U_{eff} = \sqrt{(\bar{U} + u)^2 + v^2} \qquad (3a)$$

The error caused by this assumption is discussed below. In a turbulent flow not only the magnitude of the velocity but also its direction change with time. The instantaneous angle of attack can be decomposed into the constant part α which is the angle between the axis of the tube and the mean velocity direction, and a fluctuating portion α', which corresponds to the fluctuating velocity v. Simple trigonometric relations give

$$\left.\begin{aligned} \sin \alpha' &= \frac{v}{\sqrt{(\bar{U} + u)^2 + v^2}} \\ \cos \alpha' &= \frac{\bar{U} + u}{\sqrt{(\bar{U} + u)^2 + v^2}} \end{aligned}\right\} \qquad (4)$$

Then the sine of the instantaneous angle of attack can be expressed as

$$\sin(\alpha + \alpha') = \sin \alpha \cos \alpha' + \cos \alpha \sin \alpha'$$

$$= \frac{\bar{U} + u}{\sqrt{(\bar{U} + u)^2 + v^2}} \sin \alpha + \frac{v}{\sqrt{(\bar{U} + u)^2 + v^2}} \cos \alpha$$

In a turbulent flow if the eddies are sufficiently large compared to the diameter of the tube, then the flow can be considered as quasi-steady. Thus the flow around the tube at any instant is the same as in non-turbulent flow. Equations (1) and (2) then are valid for the instantaneous values of pressure, velocity and angle of attack, and we have

$$\frac{(\bar{P}_1 + p_1) + (\bar{P}_2 + p_2)}{\frac{1}{2}\rho} = \left[(\bar{U}+u)^2 + v^2\right]\left[A + B\sin(\alpha+\alpha') + C\sin^2(\alpha+\alpha')\right] \tag{5}$$

$$\frac{(\bar{P}_1 + p_1) - (\bar{P}_2 + p_2)}{\frac{1}{2}\rho} = D\left[(\bar{U}+u)^2 + v^2\right]\sin(\alpha+\alpha') \tag{6}$$

Substituting equation (4) into equations (5) and (6) and taking time average of both equations yields

$$\frac{\bar{P}_1+\bar{P}_2}{\frac{1}{2}\rho\bar{U}^2} = A\left(1 + \frac{\overline{u^2}}{\bar{U}^2} + \frac{\overline{v^2}}{\bar{U}^2}\right) + B\left\{\overline{\left(1+\frac{u}{\bar{U}}\right)\left[\left(1+\frac{u}{\bar{U}}\right)^2 + \frac{v^2}{\bar{U}^2}\right]^{\frac{1}{2}}}\sin\alpha + \overline{\frac{v}{\bar{U}}\left[\left(1+\frac{u}{\bar{U}}\right)^2 + \frac{v^2}{\bar{U}^2}\right]^{\frac{1}{2}}}\cos\alpha\right\}$$
$$+ C\left\{\left(1+\frac{\overline{u^2}}{\bar{U}^2}\right)\sin^2\alpha + 2\frac{\overline{uv}}{\bar{U}^2}\sin\alpha\cos\alpha + \frac{\overline{v^2}}{\bar{U}^2}\cos^2\alpha\right\} \tag{7}$$

$$\frac{\bar{P}_1-\bar{P}_2}{\frac{1}{2}\rho\bar{U}^2} = D\left\{\overline{\left(1+\frac{u}{\bar{U}}\right)\left[\left(1+\frac{u}{\bar{U}}\right)^2 + \frac{v^2}{\bar{U}^2}\right]^{\frac{1}{2}}}\sin\alpha + \frac{v}{\bar{U}}\overline{\left[\left(1+\frac{u}{\bar{U}}\right)^2 + \frac{v^2}{\bar{U}^2}\right]^{\frac{1}{2}}}\cos\alpha\right\} \tag{8}$$

Using series expansions according to powers of $u/\bar{U}$ and $v/\bar{U}$ and keeping up to the third-order terms, equations (7) and (8) become

$$\frac{\bar{P}_1+\bar{P}_2}{\frac{1}{2}\rho\bar{U}^2} = A\left(1+\frac{\overline{u^2}}{\bar{U}^2} + \frac{\overline{v^2}}{\bar{U}^2}\right) + B\left(1+\frac{\overline{u^2}}{\bar{U}^2} + \frac{\overline{v^2}}{2\bar{U}^2} - \frac{\overline{u^3}}{2\bar{U}^3}\right)\sin\alpha$$

$$+\left(\frac{\overline{uv}}{\bar{U}^2}+\frac{\overline{v^3}}{2\bar{U}^3}\right)\cos\alpha\Bigg\} + C\left\{\left(1+\frac{\overline{v^2}}{\bar{U}^2}-\frac{\overline{v^2}}{\bar{U}^2}\right)\sin\alpha\right.$$

$$\left. + 2\,\frac{\overline{uv}}{\bar{U}^2}\sin\alpha\cos\alpha+\frac{\overline{v^2}}{\bar{U}^2}\right\}, \qquad (9)$$

$$\frac{\bar{P}_1-\bar{P}_2}{\frac{1}{2}\,\bar{U}^2} = D\left\{\left(1+\frac{\overline{u^2}}{\bar{U}^2}+\frac{\overline{v^2}}{2\bar{U}^2}-\frac{\overline{u^3}}{2\bar{U}^3}\right)\sin\alpha+\left(\frac{\overline{uv}}{\bar{U}^2}+\frac{\overline{v^3}}{2\bar{U}^3}\right)\cos\alpha\right\} \qquad (10)$$

Since equations (1) and (2) do not hold for large value of θ (more than about 35 degrees), the instantaneous angle $\alpha+\alpha'$ cannot be too large. Thus the turbulence level cannot be too high because α' is associated with the velocity fluctuation. Consequently α must be limited to small angles, but to obtain satisfactory results it is necessary to change α up to 20 degrees. Nevertheless, we may approximate by $\cos\alpha = 1 - \frac{1}{2}\sin^2\alpha$. Usually $\overline{u^3}/\bar{u}^3$ and $\overline{v^3}/\bar{u}^3$ are very small compared to $\overline{u^2}/\bar{u}^2$, $\overline{v^2}/\bar{u}^2$ and $\overline{uv}$, and these terms equal zero when the turbulence velocity has a Gaussian distribution.

Neglecting third-order terms and setting $\cos\alpha = 1 - \frac{1}{2}\sin^2\alpha$ in equations (9) and (10), the relationships between the mean pressure and the mean angle of attack can be expressed in the form of eqs. (1) and (2). We write

$$\frac{\bar{P}_1+\bar{P}_2}{\frac{1}{2}\,\rho\bar{u}^2} = A' + B'\sin\alpha + C'\sin^2\alpha$$

$$\frac{\bar{P}_1-\bar{P}_2}{\frac{1}{2}\,\rho\bar{u}^2} = D'\sin\alpha + E' \quad ,$$

$$\left.\begin{aligned}
A &= A \left(1 + \frac{\overline{u^2}}{\bar{U}^2} + \frac{\overline{v^2}}{\bar{U}^2}\right) + B\,\frac{\overline{uv}}{\bar{U}^2} + c\,\frac{\overline{v^2}}{\bar{U}^2} \\
B &= B \left(1 + \frac{\overline{u^2}}{\bar{U}^2} + \frac{\overline{v^2}}{2\bar{U}^2}\right) + 2C\,\frac{\overline{uv}}{\bar{U}^2} \\
C &= C \left(1 + \frac{\overline{u^2}}{\bar{U}^2} - \frac{\overline{v^2}}{\bar{U}^2}\right) - \frac{B}{2}\,\frac{\overline{uv}}{\bar{U}^2} \\
D &= D \left(1 + \frac{\overline{u^2}}{\bar{U}^2} + \frac{\overline{v^2}}{2\bar{U}^2}\right) \\
E &= D\,\frac{\overline{uv}}{\bar{U}^2}
\end{aligned}\right\} \qquad (11)$$

These are basic equations for computing turbulence characteristics $\overline{u^2}$, $\overline{v^2}$, and $\overline{uv}$.

In his paper Jezdinsky[(1)] obtained different equations, and the value of $\overline{uv}$ given by his equations is larger than that computed from equations (11). The main reason for this difference if the following.

In deriving the instantaneous relations Jezdinsky divided the term sin θ into the average value sin α, which corresponds to the angle between the mean velocity vector and the axis of the probe, and the fluctuating part (sin α)′. This is not correct because taking the average of equation (4) yields

$$\overline{\sin(\alpha + \alpha')} = \left(1 - \tfrac{1}{2}\,\frac{\overline{v^2}}{\bar{U}^2}\right) \sin\alpha - \frac{\overline{uv}}{\bar{U}^2} \cos\alpha$$

Thus the average part sin α does not correspond to α. Actually, Jezdinsky set $\overline{\sin(\alpha + \alpha')} = \sin\alpha$, and the error caused by this assumption is

$$\sin(\alpha + \alpha') - \sin\alpha = -\tfrac{1}{2}\frac{\overline{v^2}}{\overline{U}^2}\sin\alpha - \frac{\overline{uv}}{\overline{U}^2}\cos\alpha$$

Since this expression contains the turbulence characteristics in which we are interested, this error gives significant difference to final results.

Since equations (11) are derived in a rigorous manner, they are better than Jezdinsky's equations. In Chapter IV his data are compared with the present results.

Since there are more equations than unknowns, accuracy of these values can be checked. After many experiments it was found that calibration constants contain at most 1% error, as is shown in the next section, and coefficients A', B', .. E' have about 2% error because of the correction procedure which is also explained below. To calculate turbulence characteristics the ratio of both coefficients is needed, and from the accuracy of both coefficients 3% error is expected in their ratio. Under this situation the accuracy of the pressure-probe measurement can be estimated. For simplicity consider the turbulence in which $\overline{u^2} = \overline{v^2}$ and $\overline{uv} = 0$. If the actual turbulence intensity is 30%, then the measured value has about 12% of error, and if the intensity is 10%, the estimated error is 74%. But this is not quite right, because the quantity u' can be calculated from three equations, and by taking average more accurate value can be obtained if there is no systematic error. Moreover if the fluctuation becomes small, the measuring error caused by turbulence decreases and the accuracy is improved. But this instrument is not suitable for measurements in a turbulent flow whose relative intensity is less than 10%.

C. Calibration

After check was made for leakage of air and for the symmetry of both probes, the probes were calibrated in a potential core of a turbulent jet from a nozzle. The turbulence level in this core was 0.46%, and the velocity was kept constant during the calibration (the fluctuation of the velocity was less than 0.5%). Since calibration constants are slightly affected by the velocity, the probes were calibrated at four different speeds at the same point. One of the results for 0.028" probes is shown in figure (3).

The calibration constants of this probe are

$$A = 1.04 \pm 0.01$$

$$B = 0.010 \pm 0.001$$

$$C = 1.30 \pm 0.01$$

$$D = 2.64 \pm 0.00$$

These constants were determined from data by using the method of least squares. They can be determined within 1 per cent error except B, which is affected by the error in the misalignment of the probe in the flow direction. In general the coefficient A slightly increases, and coefficients C and D decrease with increasing velocity. When the velocity is small (less than 25 feet/sec for this probe), the linear relation for the pressure difference does not hold except for small angles of attack. Thus this pressure probe cannot be used with a reasonable accuracy in the low speed flow, and this is one of the drawbacks of this method compared to a hot-wire anemometer.

D. Measurements in a Turbulent Flow.

Since this pressure probe method requires very accurate measurements of pressure and velocity, care must be taken in a turbulent flow about the difference between the measured value and the true value. In general, measured values are affected by turbulence, and it is impossible to know true values. However, experimentally or theoretically the effect of turbulence can be estimated, and corrections can be made. There are some other problems in using this instrument.

1. In wall turbulence whose structure is directly influenced by a solid boundary, the mean flow direction can be found easily like a turbulent flow in a pipe. However, in free turbulence where there is no direct effect of any fixed boundary it is not a simple matter to determine the mean flow direction very accurately. Since the usual flow-direction meter is based on the same principle as this instrument, its indication is affected by both the turbulent intensity and the turbulent shear stress so that it cannot be used in a turbulent flow. In a round free jet the flow direction may be determined by measuring carefully the velocity distributions at some sections of the flow in order to discover the apparent origin of the jet. As may be seen in equations 99) and (10), the constants B' and E' are influenced by the error in the flow angularity, hence the turbulent shear

stress calculation is also affected.

2. When this pressure probe is used in a turbulent flow the mean pressure readings give

$$\bar{P}_{meas} = \bar{P} + \overline{Ps} + \Delta Pw, \qquad (12)$$

where $\bar{P}$ is the actual mean value of pressure which must be used in calculations. $\overline{Ps}$ is the mean static pressure and must be measured separately. The measurements of static pressure in turbulence are discussed in the next chapter. ΔPw represents the effect of the third velocity component. In deriving the basic equation (11), the problem was treated two-dimensionally and the velocity component which is normal to a longitudinal plane was neglected by assuming that this lateral velocity does not give an impact pressure on the probe. If the tip of the probe is infinitely small and can be considered a true point, then the lateral velocity will produce an impact pressure. However, because of the finite diameter of the probe it will rather produce a negative suction pressure. ΔPw may be small, but since the sum of pressures measured by two tubes is needed, the error becomes $2\Delta Pw$, and the correction is necessary to get better results. This improvement can be made by using the directional sensitivity of this probe.

When the probe is put in a steady flow, the relation between the yaw angle in the lateral direction and the pressure readings can be roughly approximated by

$$P_\beta - P_o = K \sin^2\beta \quad \tfrac{1}{2} \rho U^2 \qquad (13)$$

For not too large values of β and sufficiently small angles of attack it is found that $k \approx -\frac{1}{2}$. Actually K slightly depends on the size and the shape of the probe. Now if it is assumed that this relation holds instantaneously in a turbulent flow, U and β in equation (13) should be replaced by

$$U_{eff} = (\bar{U} + u)^2 + v^2 + w^2$$

$$\sin \beta = \frac{w}{U_{eff}}$$

Substituting these relations into equation (13), and taking the time average of the equation yield

$$\bar{P}_\beta - \bar{P}_o = \tfrac{1}{2}\rho K w^2$$

Noting that $\Delta Pw = \bar{P}_\beta - \bar{P}_o$, this relation gives

$$\frac{\Delta Pw}{\frac{1}{2}\rho\bar{U}^2} \approx K \frac{\overline{w^2}}{\bar{U}^2} \tag{14}$$

It is clear that the third velocity component indeed produces negative pressure.

3. There are some methods for measuring the mean velocity in turbulence, but most of them are not simple. A Pitot-tube is frequently used, but here not only the static pressures but also the total-head readings are affected by turbulence. Thus they must be corrected to get the true mean flow velocity. This correction is discussed in the next chapter.

CHAPTER III MEASUREMENTS OF TOTAL AND STATIC PRESSURES IN TURBULENCE

A. Static Pressure in a Turbulent Flow

1. Pressure Distribution in a Round Free Jet

For a steady, incompressible, non-viscous flow the equation of motion in r direction is given in (r,e,x) coordinates by

$$U_r \frac{\partial U_r}{\partial r} + \frac{U_\theta}{r} \frac{\partial U_r}{\partial \theta} - \frac{U_\theta^{\,2}}{r} + U_x \frac{\partial U_r}{\partial x} = - \frac{1}{\rho} \frac{\partial P}{\partial r} \qquad (15)$$

In a turbulent flow this equation must hold at any instant and also on the average. Each quantity is assumed to be divided into a mean part and a fluctuating part, and we write

$$U_r = \bar{U}_r + u_r, \quad U_\theta = \bar{U}_\theta + u_\theta, \quad U_x = \bar{U}_x + u_x$$

$$P = \bar{P} + p$$

Substituting these expressions into equation (15) applying the averaging procedure and using the continuity equation for turbulent velocities, that is

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial u}{\partial x} = 0,$$

yield

$$\bar{U}_r \frac{\partial \bar{U}_r}{\partial r} + \frac{\bar{U}_\theta}{r} \frac{\partial \bar{U}_r}{\partial \theta} - \frac{\bar{U}_\theta^{\,2}}{r} + \bar{U} \frac{\partial \bar{U}_r}{\partial x} = - \frac{1}{\rho} \frac{\partial \bar{P}}{\partial r} - \frac{\partial}{\partial r} \overline{u_r^{\,2}}$$

$$- \frac{\partial}{\partial x} \overline{u_r u_x} - \frac{1}{r} \frac{\partial}{\partial \theta} \overline{u_r u_\theta} - \frac{\overline{u_r^{\,2}}}{r} + \frac{\overline{u_\theta^{\,2}}}{r}$$

For an axially symmetric flow where $\bar{U}_\theta = 0$ and $\frac{\partial}{\partial\theta} = 0$, this equation reduces to

$$\bar{U}_r \frac{\partial \bar{U}_r}{\partial r} + \bar{U}_x \frac{\partial \bar{U}_r}{\partial x} = -\frac{1}{\rho}\frac{\partial \bar{P}}{\partial r} - \frac{\partial}{\partial r}\overline{u_r^2} - \frac{\partial}{\partial x}\overline{u_r u_x} - \frac{\overline{u_r^2}}{r} + \frac{\overline{u_\theta^2}}{r} \quad (16)$$

$$\delta\,\frac{1}{\delta} \qquad 1\,\frac{\delta}{1} \qquad \frac{\Delta P}{\rho\delta} \qquad \frac{\varepsilon^2}{\delta} \qquad \frac{R_{rx}\varepsilon^2}{1} \qquad \frac{\varepsilon^2}{\delta} \qquad \frac{\varepsilon^2}{\delta}$$

The center line of a circular jet is chosen as the x axis, and it is assumed that the mean velocity normal to the main flow is very small compared to the main flow velocity. Then the main flow velocity $\bar{U}x$ and the length x are assumed to be of unit order, and $\bar{U}_r$ is taken to be of the order of magnitude δ where $\delta << 1$. It follows from the continuity equation that the length r is of order δ. It is known that $\overline{u_x^2}$, $\overline{u_r^2}$, $\overline{u_\theta^2}$ are of the same magnitude in non-isotropic shear flows, and for them the same scale ε^2 is introduced. The magnitude of $\overline{u_r u_x}$ can be expressed as $R_{rx}\,\varepsilon^2$, where R_{rx} is the correlation coefficient. The relative order of magnitude of each term of equation (16) is shown below it. Since ε^2 may be of the order 1, both terms on the left hand side of equation (16) are an order of magnitude smaller. If R_{rx} is assumed to be not small, equation (16) reduces to

$$\frac{\partial \bar{P}}{\partial r} + \rho\,\frac{\partial}{\partial r}\overline{u_r^2} + \rho\,\frac{\overline{u_r^2}}{r} - \rho\,\frac{\overline{u_\theta^2}}{\rho} = 0 \qquad (17)$$

When integrated with respect to r, this equation becomes

$$\bar{P} + \rho\,\overline{u_r^2} + \rho\int \frac{\overline{u_r^2} - \overline{u_\theta^2}}{r}\,dr = P_o\,, \qquad (18)$$

where P_o is the pressure outside the turbulent region at x. If the distributions of turbulence intensities are known at some section, then the pressure distribution at that section can be calculated using equation (18).

2. Measurement of Static Pressure

Since we are interested in only the mean value of fluctuating pressure, a Pitot-static tube is very useful. If used with a manometer, it gives time-averaged readings of pressure because of the large inertia of the system.

Goldstein[2] assumed that the reading of the Pitot static tube differs from the true average static pressure by a pressure arising from the impact of the fluctuating cross velocities on the tube and its holes. The relation between the reading of the tube aligned in the mean flow direction and the true average static pressure can then be written in the form

$$\bar{P}_{sm} = \bar{P}_s + k\,\rho(\overline{v^2} + \overline{w^2}) \tag{19}$$

where k is a constant which depends on the design of the tube and has to be determined by experiments.

According to equation (19) Fage[3] made experiments both in a circular and in an elongated rectangular pipe, where theoretical relations between $\bar{P}_s$ and $\overline{v^2}$, $\overline{w^2}$ are known, and he determined k = 0.25 for a round-nosed Pitot tube. Then equation (19) can be written

$$\overline{P_{sm}} = \bar{P}_s + \tfrac{1}{2}\left(\frac{\overline{v^2}}{\bar{U}^2} + \frac{\overline{w^2}}{\bar{U}^2}\right)\tfrac{1}{2}\rho\bar{U}^2 \tag{20}$$

Later Toomre[4] pointed out that equation (19) is valid only if the turbulent eddies are small compared with the diameter of the static tube so that the pressure fluctuations at different holes are uncorrelated. For an ordinary Pitot static tube this situation may occur in grid turbulence. If the scale of eddies is large enough so that the pressure fluctuations are correlated at different static holes, the effect of turbulence on static pressure readings can be found by using the direction-sensitivity characteristic of a Pitot tube. From systematic experiments in a circular jet Bradshaw and Goodman[5] found that the influence of turbulence on static pressure readings indeed depends on the ratio of eddy size to tube diameter (see Figure 5). The asymptotic value of their data is close to the true static pressure computed according to equation (18). But this result depends mainly on the type of turbulence and the shape of the static tube. Therefore, it is necessary to find a correction form for each static tube.

In a turbulent flow in which the pressure probe can be used the eddy size is assumed to be much larger than the static tube diameter, and the steady yaw response can be used to correct for turbulence. For the usual round-nosed Pitot tube the equation of the steady yaw response is given by

$$P_{sm} = P_s - \ell \rho U^2 \sin^2 \theta \qquad (\theta < 20^o) \tag{21}$$

where ℓ is a constant and it depends on the size and the shape of the static tube. In Figure 6 the result of the yaw angle response of the static tube, which was used in the present experiments, is presented, and for this tube $\ell = 0.4$.

In a turbulent flow

$$\sin^2\theta = \frac{v^2 + w^2}{U_{eff}^2} = \frac{v^2 + w^2}{(\bar{U}+u)^2+v^2+w^2} \qquad (22)$$

Then equation (21) gives

$$\bar{P}_{sm} = \bar{P}_s - 2\ell\left(\frac{\overline{v^2}}{\bar{U}^2} + \frac{\overline{w^2}}{\bar{U}^2}\right) \tfrac{1}{2}\,\rho\bar{U}^2 \qquad (23)$$

Comparing equation (20) with (23) it is clear that contrary to the Goldstein - Fage argument a static tube reads too low in turbulence.

In Figure 6 the static pressure across a circular jet is shown. The results from equations (20) and (23) are compared with the theoretical value given by equation (18). In a circular jet it can be assumed $\overline{u_r^2} = \overline{u_\theta^2}$, and the integral term of equation (18) is negligible. The static pressure is measured by a Pitot tube whose yaw response is shown in Figure 6, and the values of $\overline{v^2}$ are taken from the hot wire data by Corrisin and Uberoi[6]. As can be seen in the figure, the values calculated by equation (23) show reasonable agreement with the theoretical results near the center of a jet, but away from the center line the agreement becomes poor. It seems that there are some other problems in a highly turbulent region.

If it is desired to use a Pitot tube for measuring the fluctuating static pressure in turbulence, equation (21) with $\sin^2\theta$ given by equation (22) hold instantaneously. By subtracting the average equation (23), the difference between the fluctuating parts of P_{sm}

and P_s is given by

$$P_{sm} - P_s = -0.4\rho \ (v^2 + w^2 - \overline{v^2} - \overline{w^2}).$$

Setting $v^2 + w^2 = u_t^2$, this becomes

$$P_{sm} - P_s = -0.4\ \rho \ (u_t^2 - \overline{u_t^2}) \tag{24}$$

For isotropic turbulence the mean-square error of the measurement of the fluctuating pressure is obtained from equation (24) as

$$\overline{P_{sm} - P_s}^{\,2} = 1.28 \ \left(\rho \overline{u^2}\right)^2, \tag{25}$$

where it is assumed that the lateral turbulent velocity has a Gaussian distribution such that $\overline{u_t^4} = 3\left(\overline{u_t^2}\right)^2$. Batchelor[7] derived theoretically the mean square value of the static pressure fluctuations in an isotropic turbulence. His result was

$$\overline{p^2} = 0.34 \ \left(\rho \overline{u^2}\right)^2 \tag{26}$$

Comparing equation (25) with (26) it is apparent that the estimated error is much larger than the magnitude of the pressure fluctuations. Although Strasberg[8] found that the static pressure fluctuations in shear flows are larger than equation (26), a Pitot static tube is not suitable to measure the static pressure fluctuations in turbulence.

B. Measurement of Total Pressure

Although there is no reliable investigation concerning the effect of turbulence intensities on the readings of a total-head tube, some methods are available to correct for turbulence.

Hinze and Van der Hegge Zijnen[9] neglected completely the effect of lateral velocity fluctuations, and they used the relation

$$\overline{P_{tm}} = \overline{P_s} + \tfrac{1}{2}\rho\bar{U}^2\left(1+\frac{\overline{u^2}}{\bar{U}^2}\right) \tag{27}$$

Using the directional sensitivity of a total head tube Hinze[10] obtained

$$\overline{P_{tm}} = \overline{P_s} + \tfrac{1}{2}\rho\bar{U}^2 \tag{28}$$

This relation implies that the total-head tube readings are not affected by turbulence. Actually this approximation is quite crude.

The effect of turbulence depends on the shape and the diameter of the total-head-tube hole as may be seen from the fact that the equations for the direction-sensitivity of different types of total-head tubes vary considerably, (reference (11)). For example, a very thin total head tube, whose hole diameter is 0.008 inch, is insensitive to the flow direction, and up to about 40 degrees the readings of total pressure do not change. (Figure 8). For a total-head-tube of this kind the measured total pressure can be expressed as

$$\overline{P_{tm}} = \overline{P_s} + \left(1 + \frac{\overline{u^2}}{\bar{U}^2} + \frac{\overline{v^2}}{\bar{U}^2} + \frac{\overline{w^2}}{\bar{U}^2}\right)\tfrac{1}{2}\rho\bar{U}^2 \tag{29}$$

In fact, from a theoretical study Goldstein[2] found the same equation.

From equations (23) and (29)

$$\overline{P_{tm}} - \overline{P_{sm}} = \left[1 + \frac{\overline{u^2}}{\bar{U}^2} + 1.8\left(\frac{\overline{v^2}}{\bar{U}^2} + \frac{\overline{w^2}}{\bar{U}^2}\right)\right]\tfrac{1}{2}\rho\bar{U}^2 \tag{30}$$

This equation is not general, because both equations (23) and (29) are correct only for a special type of static and total head tubes.

CHAPTER IV MEASUREMENT OF TURBULENCE IN A ROUND FREE JET.

A. Experimental Apparatus

The sketch of a jet tunnel is shown in Figure 9. The fan is a centrifugal type, and it is driven by a three-phase induction motor which is rated at 3 horsepowers. A wooden four-inch circular nozzle is installed at the end of the tunnel, and the velocity at the mouth is 143 ft/sec, which gives $Re = 2.8 \times 10^5$. All tests were run at this velocity, and the mean velocity fluctuation during experiments was less than 1%. The turbulent level at the center of the mouth was 0.13%.

0.028-inch diameter pressure probes were used, and pressure was measured by Meriam micromanometer Model 34FB2 TM reading in thousandths of an inch of water pressure.

B. Mean Velocity Distribution

The mean velocity was measured by a thin total head tube which is very insensitive to the yaw angle and a Pitot static tube. Their readings are corrected for turbulence according to equations (23) and (29). The mean velocity measured in this way is $\bar{U}_R = \sqrt{\bar{U}^2 + \bar{V}^2}$ rather than $\bar{U}$, but the radial velocity component $\bar{V}$ is small compared to the axial component $\bar{U}$ except near the edge of a jet. For example, according to Twonsend[12] $|\bar{V}|/\bar{U} < 0.027$ for $r/r_{\frac{1}{2}} < 1.5$ so that we can set $\bar{U}_R \doteq \bar{U}$ in that region.

Figure 10 shows the distribution of the mean velocity along a center line of the jet. The length of the potential core region is about four times nozzle diameter. Beyond this point the mean velocity decreases hyperbolically with distance from the nozzle and their relation is approximately given by

$$\frac{\bar{U}_m}{U_p} = 6.4 \; \frac{D}{x} \quad (4 < x/D < 40).$$

The apparent origin seems to exist just at the exit of the nozzle.

In Figure (11) the axial distributions of the mean velocity at x = 10D and 20D are shown. They are plotted together in the same coordinates, where both axes are non-dimensionalized by the mean velocity at the center of the jet and the half-velocity radius respectively. Both profiles show good similarity, and they are very close to the Gaussian error curve.

C. Measurement of Turbulence Quantities

The turbulence quantities u', v', and $\overline{uv}$ were measured at two sections of the jet (x/D = 10, 15). The turbulence component u' measured by pressure probes is the component in the direction of $\bar{U}_R$, but as long as the distance from the axis of a jet is not large, this component is almost equal to the true axial component within the limits of accuracy of the measurements.

Measurements were limited to the region near the axis of the jet since the pressure probe cannot be used as a reliable instrument when the mean velocity is small (less than 25 ft/sec). Also in the region away from a centerline it is difficult to measure the correct static pressure because of the high turbulence level.

The results are plotted in Figures 12 to 17. To check the accuracy of pressure probes the results were compared with those taken by a hot-wire. Actually, a hot-wire anemometer was not used in the present experiment. Several data have been published, and Corrsin and Uperoi [(6), (13)] made very systematic experiments. Their data are plotted in Figures (12 to 17).

D. Experimental Procedure

The procedure of the pressure probe method is the following.

a. Calibration of the pressure probe in a nonturbulent flow.

b. In free shear turbulence, it is difficult to measure the mean flow direction accurately since its measurement is affected by turbulence. In a round free jet this can be done as follows. By finding the points of maximum total pressure at some sections the mean flow direction on the axis is determined. Beyond the section $x/D = 10$ the mean axial velocity distribution is similar and very close to the normal distribution (Figure 11). Then it is possible to compute the mean radial velocity from this curve using the continuity equation, and the mean flow direction off the axis is determined.

c. Measurements of the mean total pressure by a total head tube and the mean static pressure by a Pitot static tube.

d. Measurements of pressures at some angles by the pressure probe.

e. All these data must be corrected according to the estimated values of turbulence intensities. Then turbulence quantities can be calculated using equations (11). If necessary calculations are repeated according to calculated values . If the mean flow direction is known and it is desired to measure only turbulent shear stress, the step (d) is simple. If the difference of two pressures at zero angle of attack and the mean velocity are measured, then the value of turbulent shear stress can be easily calculated with good accuracy.

E. Discussion of Results

The distributions of turbulence intensities u' and v' measured by pressure probes show reasonable agreement with those taken by the hot-wire anemometer. The hot-wire data themselves do not coincide with each other probably because of the difference in the exit Reynolds number and the mechanical setup and also because of the measuring error. However, they follow the same tendency with increasing distance from the centerline, and the pressure probe data also follow it.

The turbulent shear stress distributions represented by one-point double correlation $\overline{uv}$ are shown in Figures 14 and 17. Theoretically shear stress can be computed from the equation of motion assuming the similar profile of the axial velocity distribution. It is given by

$$\frac{\sigma_{xr}}{\rho \bar{U}_m^2} = -\frac{1}{\eta} \frac{\bar{U}}{\bar{U}_m} \int_0^{\eta} \frac{\bar{U}}{\bar{U}_m} \eta \, d\eta \; , \quad \left(\eta \equiv \frac{r}{x}\right)$$

Since viscosity is neglected away from walls

$$\sigma_{xr} = -\rho \, \overline{uv}.$$

Then
$$\frac{\overline{uv}}{\bar{U}_m^2} = \frac{1}{\eta} \frac{\bar{U}}{\bar{U}_m} \int_0^{\eta} \frac{\bar{U}}{\bar{U}_m} \eta \, d\eta \qquad (31)$$

From the velocity profile, as shown in Figure 11, $\overline{uv}$ can be computed, and the results are plotted in Figures 14 and 17. Hot-wire data are always smaller than computed values except near the jet axis, and this error was considered to be inherent in the hot-wire method. Pressure probe data are very close to computed values.

In deriving equation (11) the third order terms $\overline{u^3}$ and $\overline{v^3}$ were neglected, because for a normal distribution they are zero. However, actually the distribution of the fluctuating velocity deviates from the normal curve except on the axis of a jet. Then $\overline{u^3}$ and $\overline{v^3}$ are not zero. These terms may become significant when $\overline{uv}$ is also very small. However, their influence can be neglected within the accuracy of the present measurements.

Jezdinsky also measured turbulence quantities in a circular jet by pressure probes, and his results are shown in Figures 12 to 14. However, as was mentioned in Chapter II, he used an erroneous equation and his data should be corrected. Jezdinsky's values of $\overline{uv}$ become much larger than computed values. $\overline{uv}$ can be computed simply and accurately from the last relation in equations (11). Since the constant E' is obtained from the relationship for the pressure difference, the influences of the lateral turbulent velocity component w and of the error in the measurement of static p pressure are cancelled. However, E' is affected by the error in the alignment of probes in the mean flow direction. Then in a free jet the incluence of the radial velocity component $\bar{V}$ is important. Assuming the similar profile of the axial velocity distribution it is possible to compute $\bar{V}$ from the equation of motion as in the shear stress calculation. $\bar{V}$ is given by

$$\frac{\bar{V}}{\bar{U}_m} = \eta \frac{\bar{U}}{\bar{U}_m} - \frac{1}{\eta} \int_o^{\eta} \frac{\bar{U}}{\bar{U}_m} \eta \, d \eta$$

Neglecting $\bar{V}$, that is, assuming the mean flow direction is parallel to the axis of the jet and aligning the probes in this direction, the values of $\overline{uv}$ were measured to be very large, as in Jezdinsky's data.

He also neglected the influence of the lateral velocity fluctuation on the pressure readings. As equation (14) shows,this factor is not significant, but it is better to take it into consideration to get good results.

In Figure 18 some of the data taken by pressure probes is presented. The measured values are corrected for turbulence according to proceedures explained in Chapter II and III. Since the corrections depend on turbulence intensities, at first it is necessary to estimate their rough values from data. After calculating intensities this procedure is repeated to get better results. Actually if one has enough experience it is not difficult to estimate reasonable values, and no iteration is necessary. In this experiment the data were taken at nine different angles of attack, as in the calibration. To obtain good results it is necessary to take much data since the problem is to find the best fitting curve from data. However, much time is required, and considering the accuracy of measurements, there is some limit. Since we are interested in the time-mean value of the fluctuating pressure, an averaging device is necessary. However, because of the large inertia of the measuring system, fluctuations are damped out, and the mean value can be obtained without any extra instrument unless very large scale turbulence is involved. A problem arises in that mean readings require 7 ~ 10 minutes depending on the measuring technique and the scale of turbulence. Since we need not only the difference but also the sum of the pressure readings of two tubes, they must be measured separately, and this causes some measuring errors. In the present experiments the approximate time to take one set of data was as follows.

measurements of static and total pressure	0.4 hours
measurements by pressure probes (9 points)	2.5
calculation	0.5
Total	3.4 hours

Efforts have been made to decrease the number of measuring points, and it was found that if the turbulence level is very high (more than 25%) 5 points are enough to get reasonable results. 6 ~ 7 points are necessary in a turbulent flow of lower level.

The most reliable method to determine the coefficients from data is the method of least squares. Although this technique involves many calculations, a computer can be used if necessary.

CHAPTER V. SUMMARY, CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK.

A. Summary

In order to measure turbulent intensity and shear stress a pressure probe was developed. Originally Jezdinsky[1] made the same instrument, but the equation on which his interpretation of the measurements is based was incorrect because of his rough approximation. It gives one-half the value of the true shear stress. An improved equation is given by equation (11). The probe was assumed to be sensitive to the velocity fluctuations u and v, but relatively insensitive to w. Actually the third velocity component produces a negative pressure, and the mean pressure readings of the probe are lower than the true values. Jezdinsky neglected the effect of this component, but this effect becomes important in a highly turbulent flow. The correction form for this velocity fluctuation was obtained by making use of the directional sensitivity of the pressure probe. Since the mean pressure reading gives the sum of the pressure due to the total velocity, which is needed for calculations and static pressure, it is necessary to measure the static pressure separately. If a Pitot static tube is used in a turbulent flow, the static pressure is not measured correctly, because the pressure in static holes is affected by turbulent velocities. A correction is necessary to get the true value. Two correction forms were discussed. It was found that in a free jet the value corrected according to equation (23) agrees with the theoretical value. A thin total-head tube was used to measure the mean velocity, and its readings were corrected for turbulence according to equation (29). The instrument was tested in a free circular jet, and the results were compared with the hot-wire data. The agreement was satisfactory.

B. Conclusions

Pressure probes can be used as an instrument to measure both the longitudinal turbulence intensity u' and lateral intensities v' and w' with reasonable accuracy when these values are not small (more than 10%). Also, this instrument can measure the cross correlations $\overline{uv}$ and $\overline{uw}$ in a turbulent shear flow. In using pressure probes the following limitations to this method must be kept in mind.

1. The calibration curve must be independent of the Reynolds number based on the diameter of the probe. However, if the mean flow velocity is very small, the effect of the Reynolds number becomes significant, and the probe can no longer be used. Thus, below a Reynolds number of about 700 in air, this probe is not reliable.
2. If the velocity fluctuations are small compared to the accuracy of the measurements made by pressure probes, the data are not reliable. The lower limit of the fluctuation level is about 10%. On the other hand, if the fluctuation level is very high and the instantaneous angles of attack often surpass the maximum angle above which the steady calibration curve does not hold, this method is not applicable.
3. Both the length and time scales of energy containing eddies are important factors. In order to get the quasi-steady condition

around the tip of the probe, the length scale of turbulence must be large compared to the size of the probe. Since the diameter of the probe is very small, usually this condition is satisfied. The average value of the fluctuating pressure is defined by

$$\bar{P} = \lim_{T \to \infty} \frac{1}{T} \int_{T_1}^{T_1 + T} P(t)\, dt$$

Practically, the averaging time T cannot be made infinitely long, but T at least must be sufficiently large compared to the time scale of the fluctuations. However, when the inertia of the measuring system is very large, the time required to get the mean readings is much larger than this scale.

4. The accuracy of the pressure probe method is estimated to be less than 10% if $u'/\bar{U} > 0.2$. As for the shear stress measurement, the data are always larger than the corresponding hot-wire data, but they are closer to the theoretical values. Owing to the nature of basic equations, a small measuring error greatly affects the final results. Then the scatter of the value u' is about 5% when $u'/\bar{U} = 0.25$, and that of $\overline{uv}$ is 3%.

C. Suggestions for future work

Although it was found that this instrument can be used to measure turbulence intensity and shear stress, there are some problems which have to be investigated in the future:

1. Although the flow near the tip of the pressure probe is not known, one can expect vortex generation and shedding there. Thus, it is important to know the effect of the flow pattern

around the tip on pressure readings. It is difficult to discuss theoretically, but experimentally this may be onde, for example, by the flow visualization method.

2. In the present experiments a micromanometer was used to measure pressure accurately, but it takes a long time to get the mean value because of the inevitably large inertia. This situation can be improved by using a small microphone which is sensitive up to thousandths of an inch of water pressure, and transduces the pressure readings to electric signals. In this case it would be possible to measure the pressure sum and the pressure difference at the same time. Also, by this microphone it is possible to detect the pressure fluctuation. By checking the correlation between u,v, and p, one can get information about the behavior of the pressure probe.

3. The accuracy of this pressure probe depends on the magnitude of the turbulent intensity. The relation between them may be obtained by vibrating the pressure probe in the longitudinal and the normal direction in a relatively low turbulence flow. Since the flow speed cannot be small, we must use a high frequency vibrator. This method causes some mechanical troubles. For example, when the uniform flow velocity is 30 ft/sec (this is nearly the lower limit for this probe) and if it is desired to make a 20% fluctuation by the sinusoidal vibration of the probe the frequency must be 65 cycles/sec for a half inch stroke motion.

LIST OF REFERENCES

1. Jezdinsky, V.: "Measurement of Turbulence by Pressure Probes", AIAA Journal, Vol. 4, 1966.

2. Goldstein, S.: "A Note on the Measurement of Total Head and Static Pressure in a Turbulent Stream", Proc. Roy. Soc., London, Series A. Vol. 155, 1936.

3. Fage, A.: "On the Static Pressure in Fully-Developed Turbulence Flow", Proc. Roy. Soc., London, Series A, Vol. 155, 1936.

4. Toomre, A.: "The Effect of Turbulence on Static Pressure Measurements", Aeronautical Research Council 22010, 1960.

5. Bradshaw, P., and Goodman, D.G.: "The Effect of Turbulence on Static Pressure Tubes", Aeronautical Research Council R. & M., No. 3527, 1966.

6. Corrsin, S. and Uberoi, M.S.: "Further Experiments on the Flow and Heat Transfer in a Heated Turbulent Air Jet", NACA TN 1865, 1949.

7. Batchelor, G.K.: "Pressure Fluctuations in Isotropic Turbulence", Proc. Cambr. Phil. Soc., Vol. 47, 1951.

8. Strasberg, M.: "Measurements of the Fluctuating Static and Total-Head Pressures in a Turbulent Wake", David Taylor Model Basin, Report 1779, 1963.

9. Hinze, J.O., and B.G. van der Hegge Ziznen: "Transfer of Heat and Matter in the Turbulent Mixing Zone of an Axially Symmetric Jet", Appl. Sci. Research, 1A, 1949.

10. Hinze, J.O.: Turbulence, An Introduction to Its Mechanism and Theory, McGraw-Hill, New York, 1959.

11. Davies, P.O.A.L.: "The Behavior of a Pitot Tube in Transverse Shear Flow", Jour. of Fluid Mech., Vol.3, 1957.

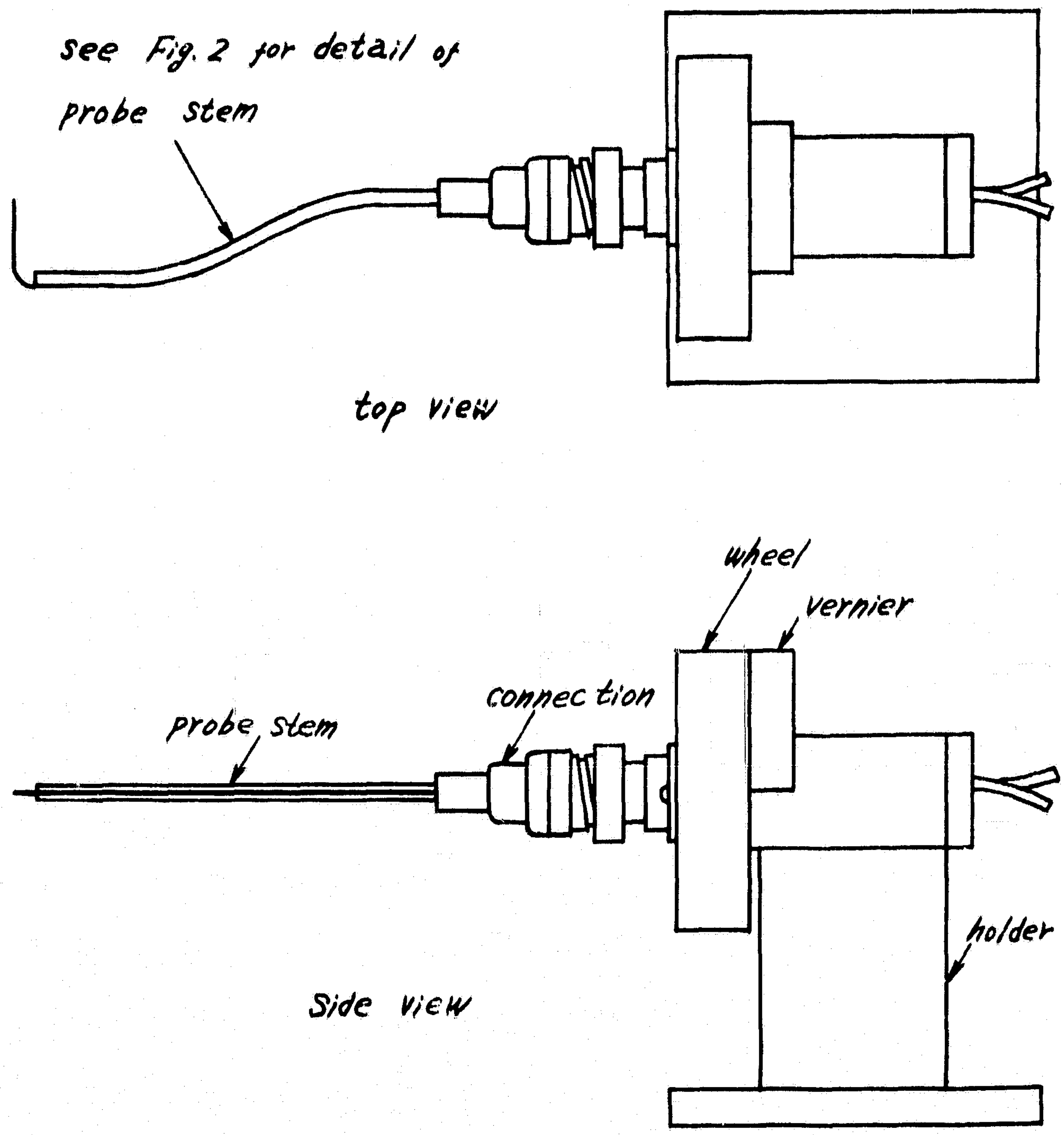
LIST OF REFERENCES-CONT'D

12. Townsend, A.: The Structure of Turbulent Shear Flow, Cambr. Univ. Press, 1956.

13. Corrsin, S.: "Investigation of Flow in an Axially Symmetrical Heated Jet of Air", NACA Wartime Reports, W94, 1943.

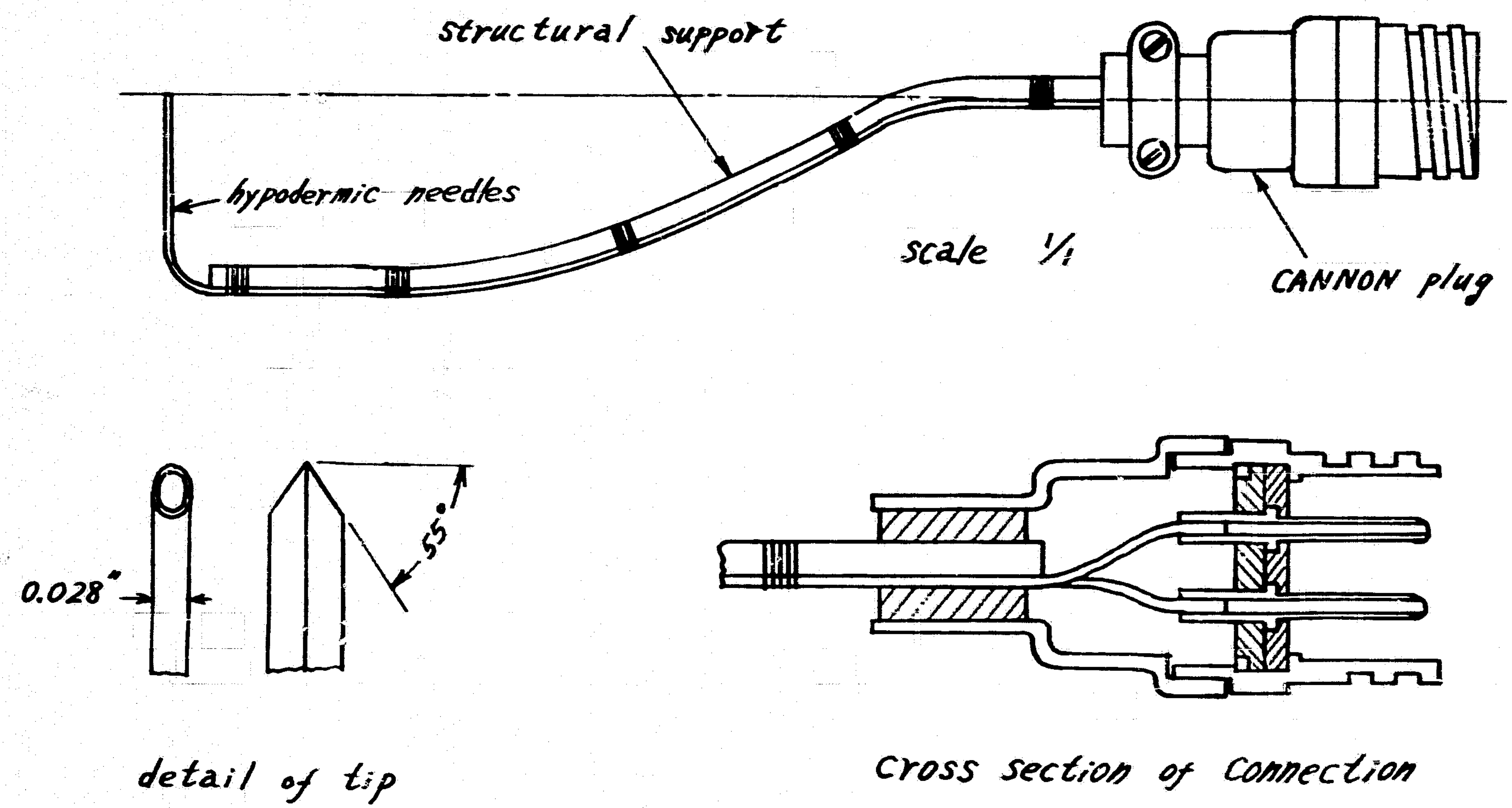
14. Albertson, M.L., Dai, Y.B., Jensen, R.A., and Rouse, H.: "Diffusion of Submerged Jets", Am. Soc. of Civil Eng., Transactions, No. 2409, Vol. 115, 1950.

15. Bradshaw, P., Ferriss, D.H., and Johnson, R.F.: "Turbulence in the Noise-Producing Region of a Circular Jet", Jour. of Fluid Mech., Vol. 19, 1964.



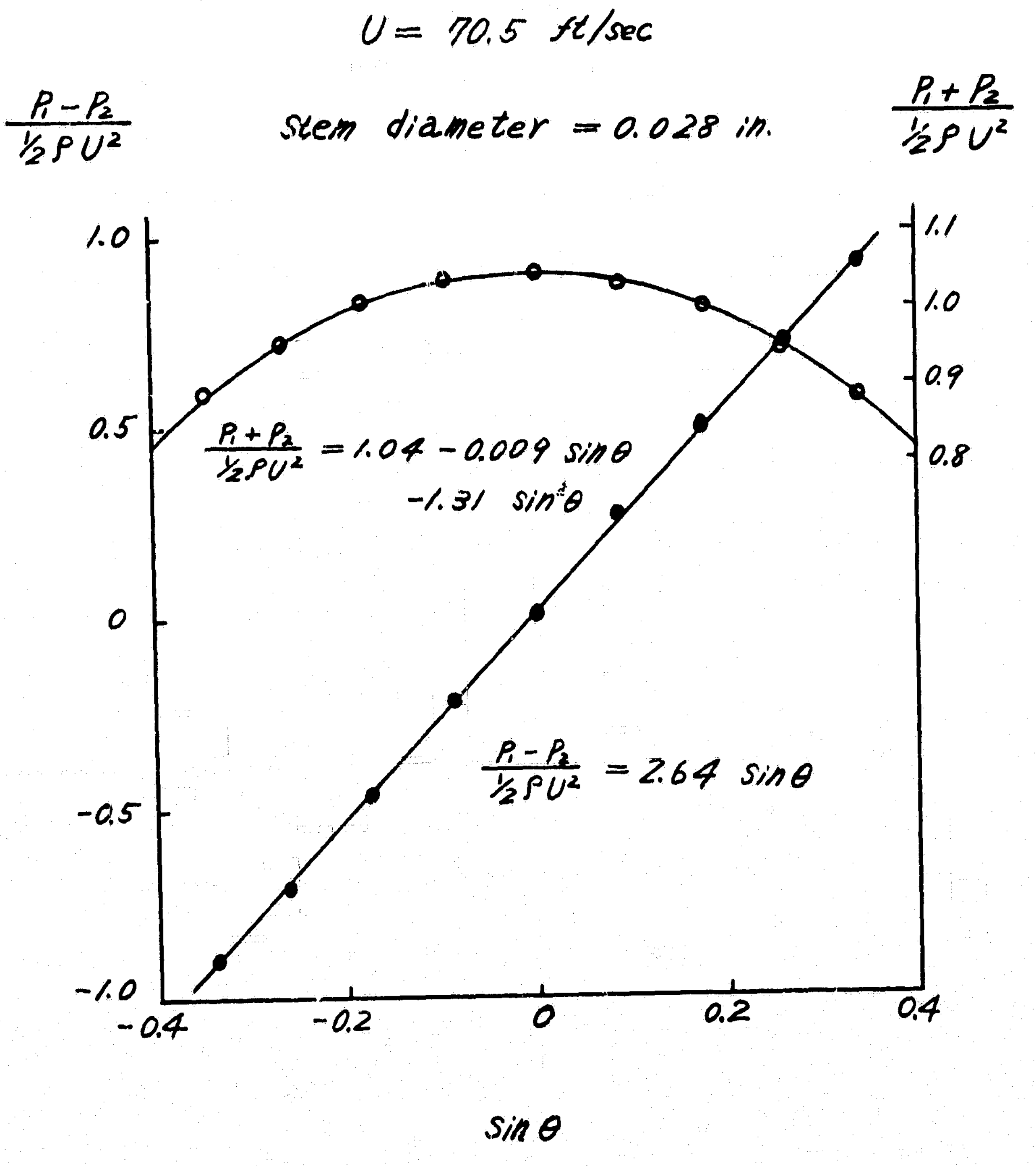
Sketch of Pressure Probes

Figure 1



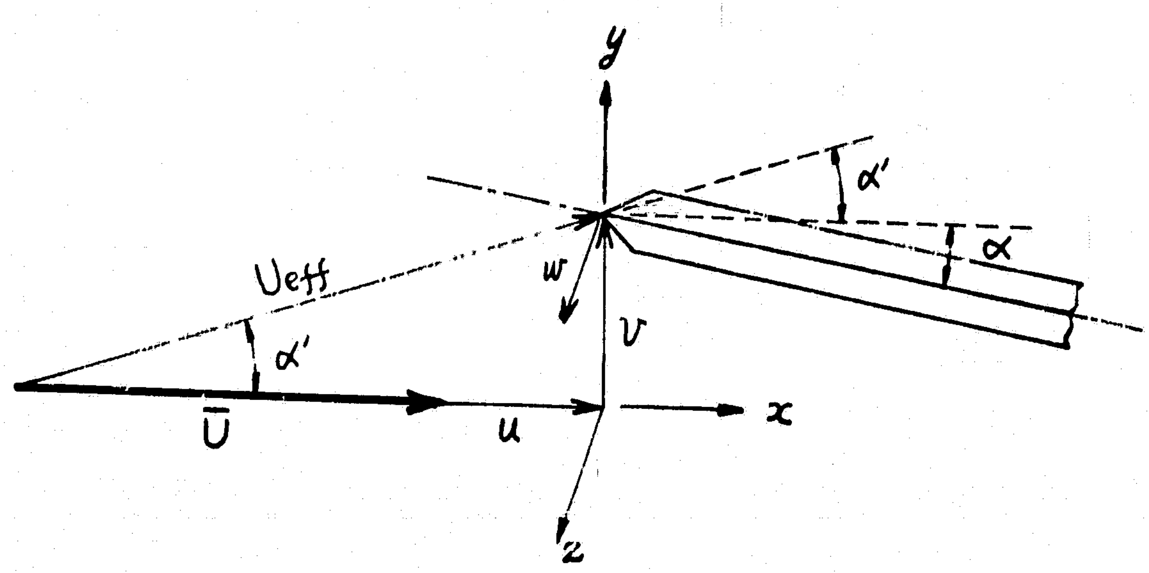
Detail of Probe Stem

Figure 2



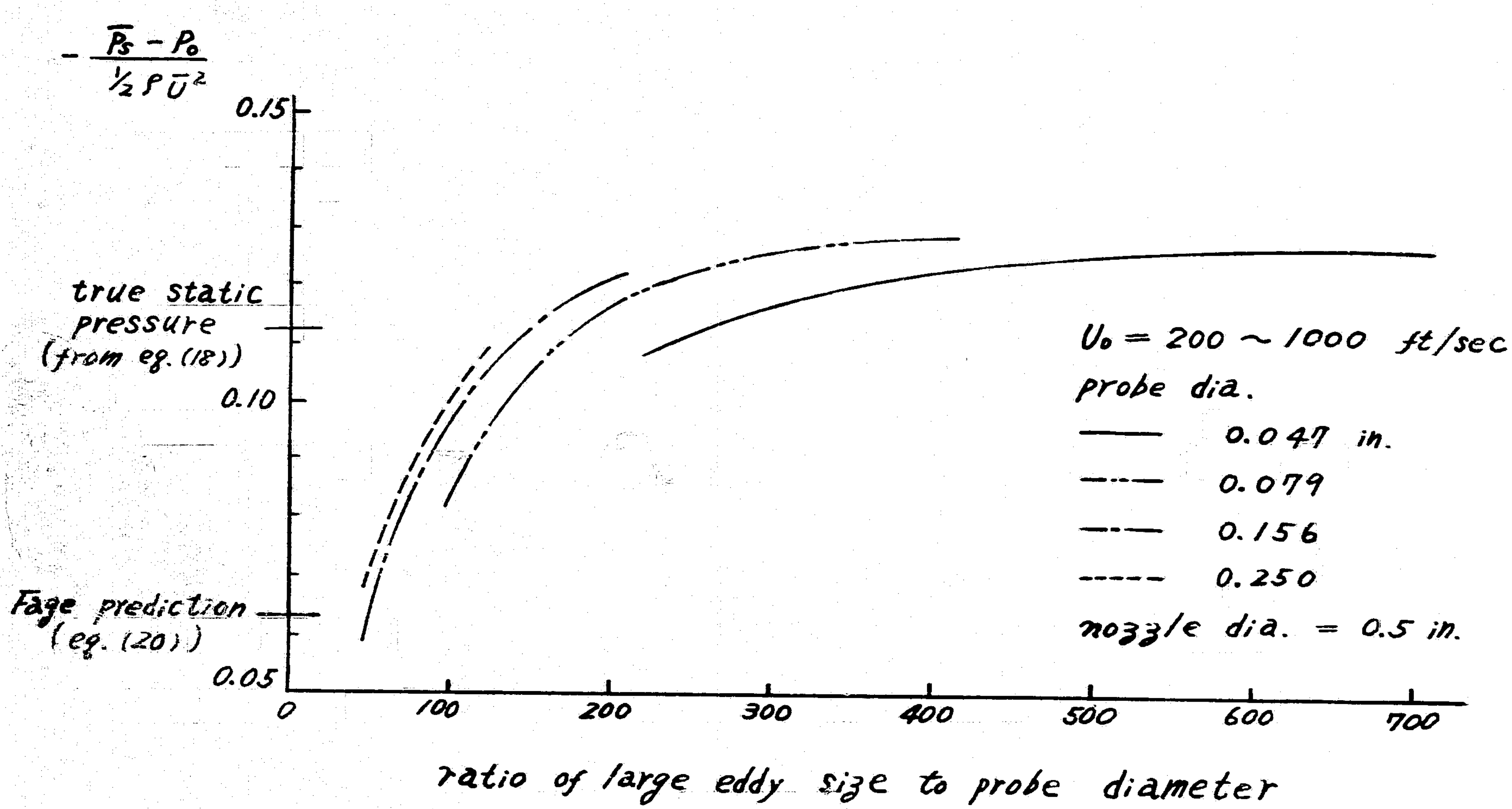
Calibration Curves of Pressure Probes

Figure 3



Coordinate System of Pressure Probes

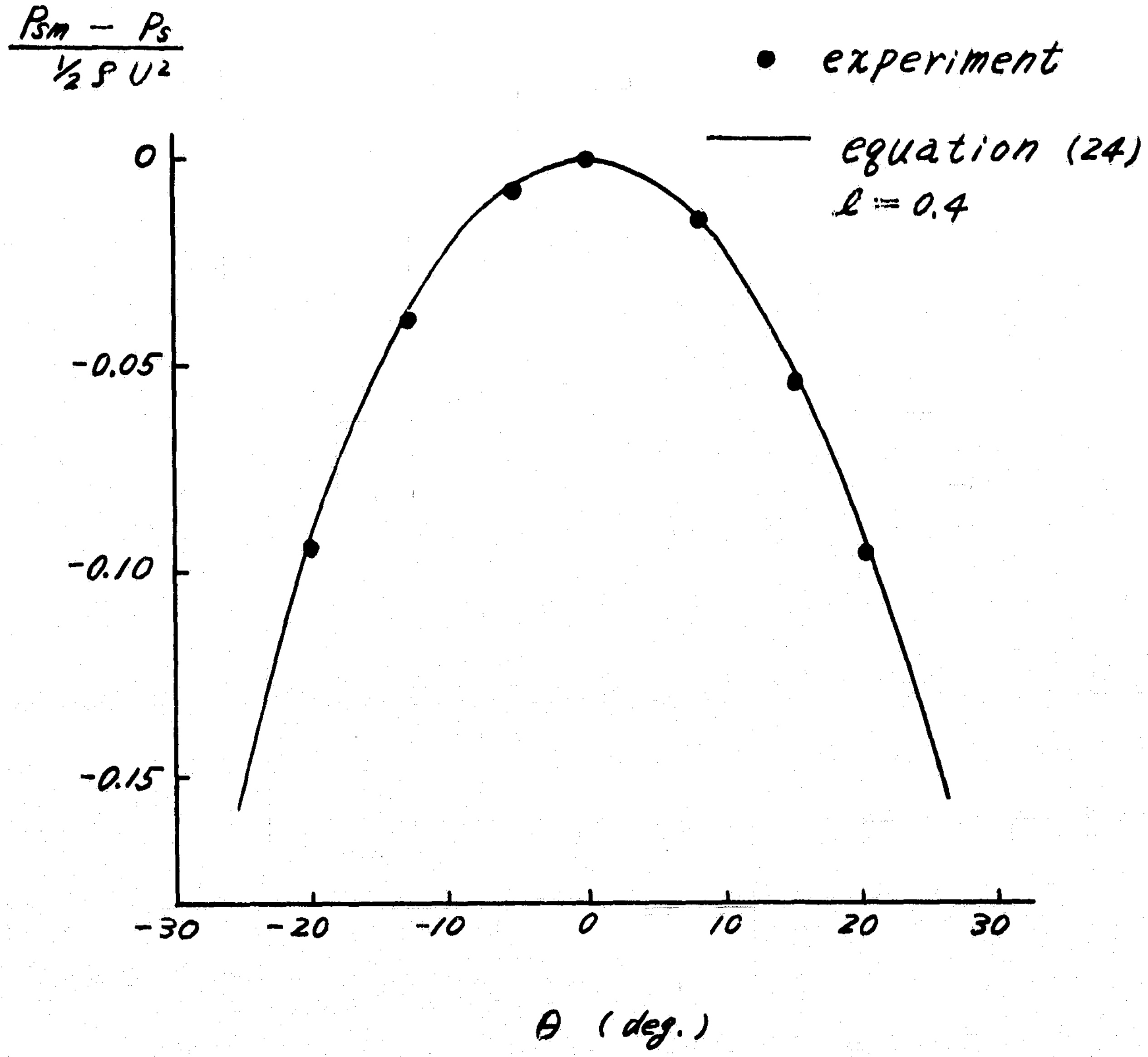
Figure 4



Measurements of Static Pressure along Centerline of Jet (from Ref. (5))

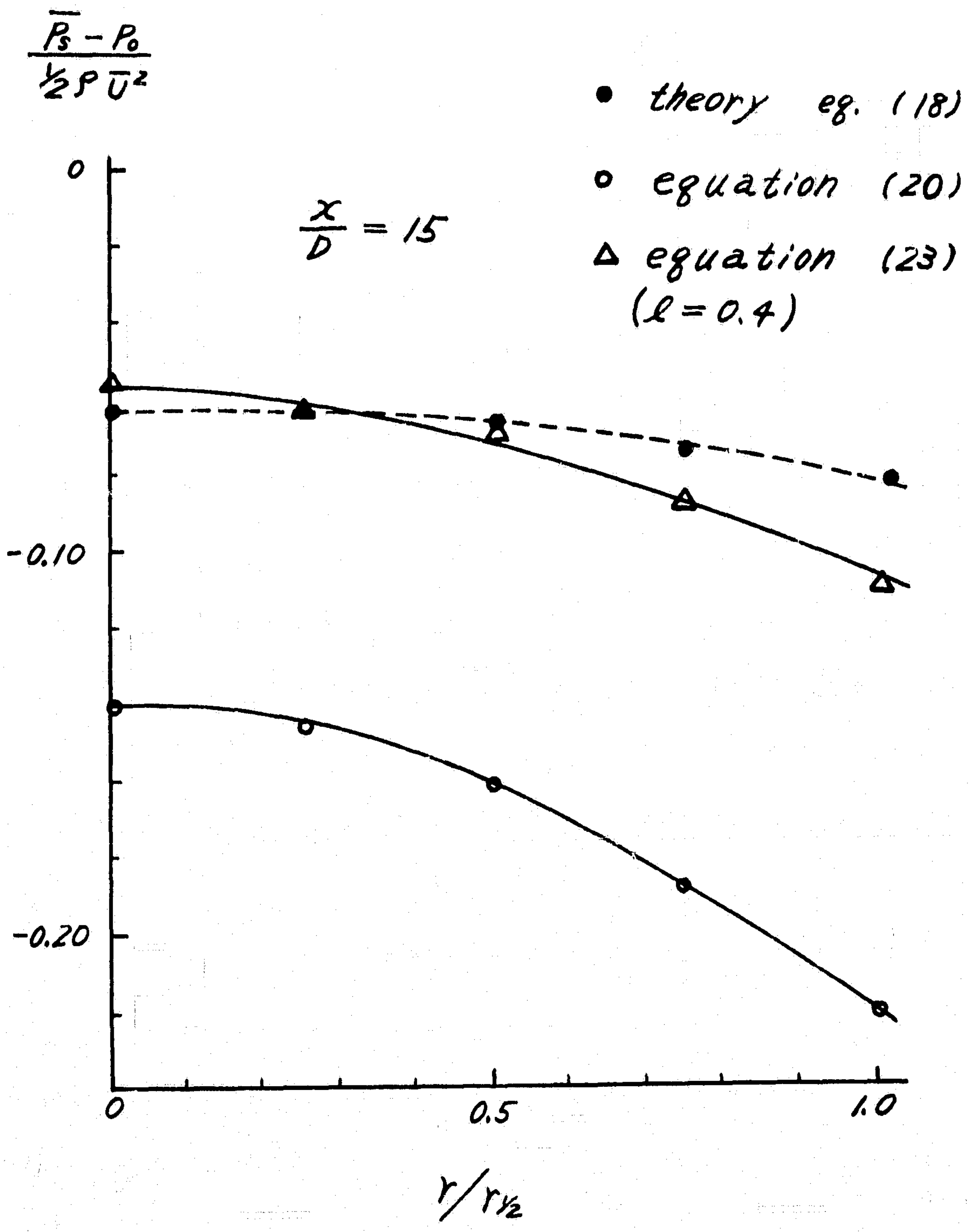
Figure 5

tube diameter 0.12 in.
hole diameter 0.010 in.
4 holes



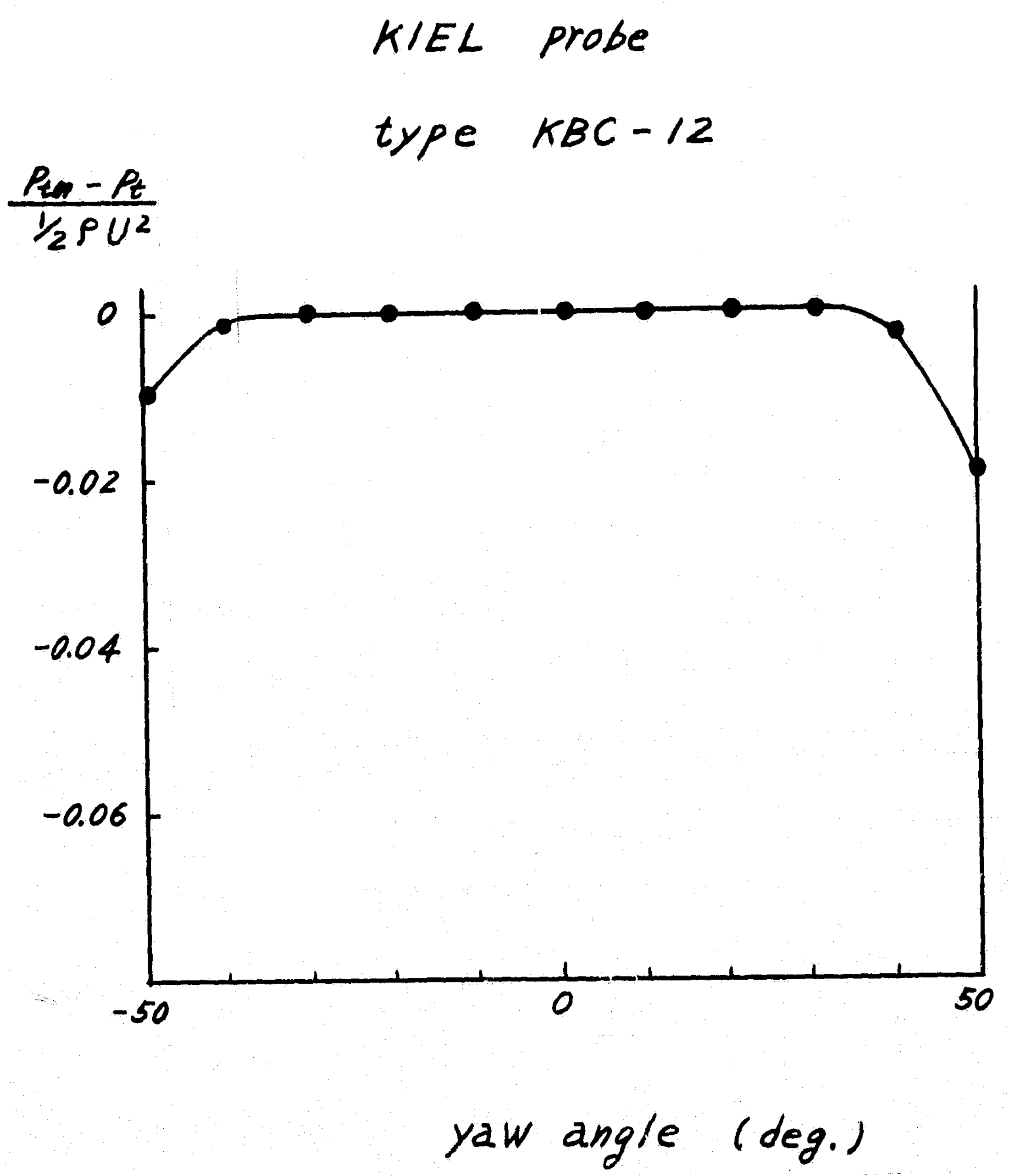
Steady Yaw Response of a Pitot Static Tube

Figure 6



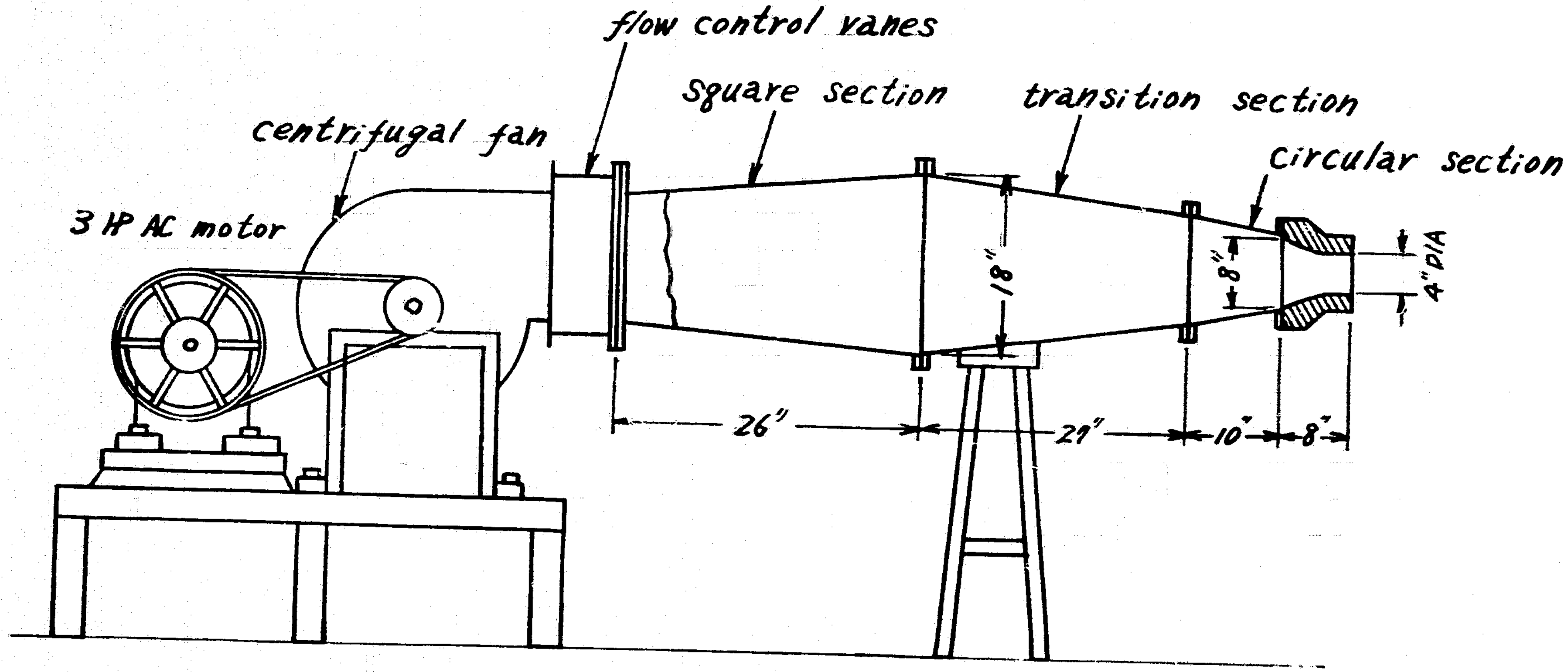
Pressure Distribution across a Jet

Figure 7



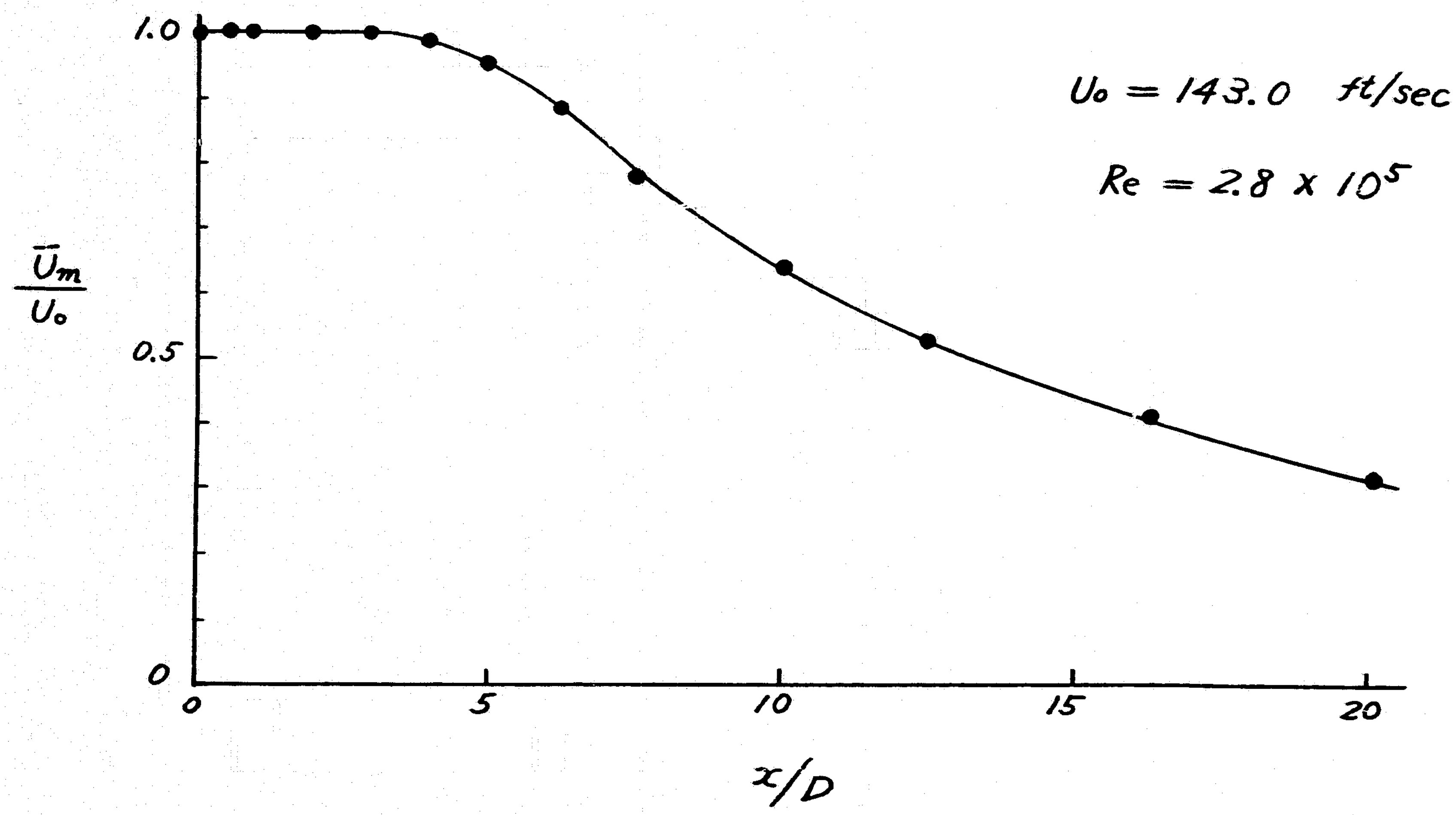
Steady Yaw Response of a Total Head Tube

Figure 8



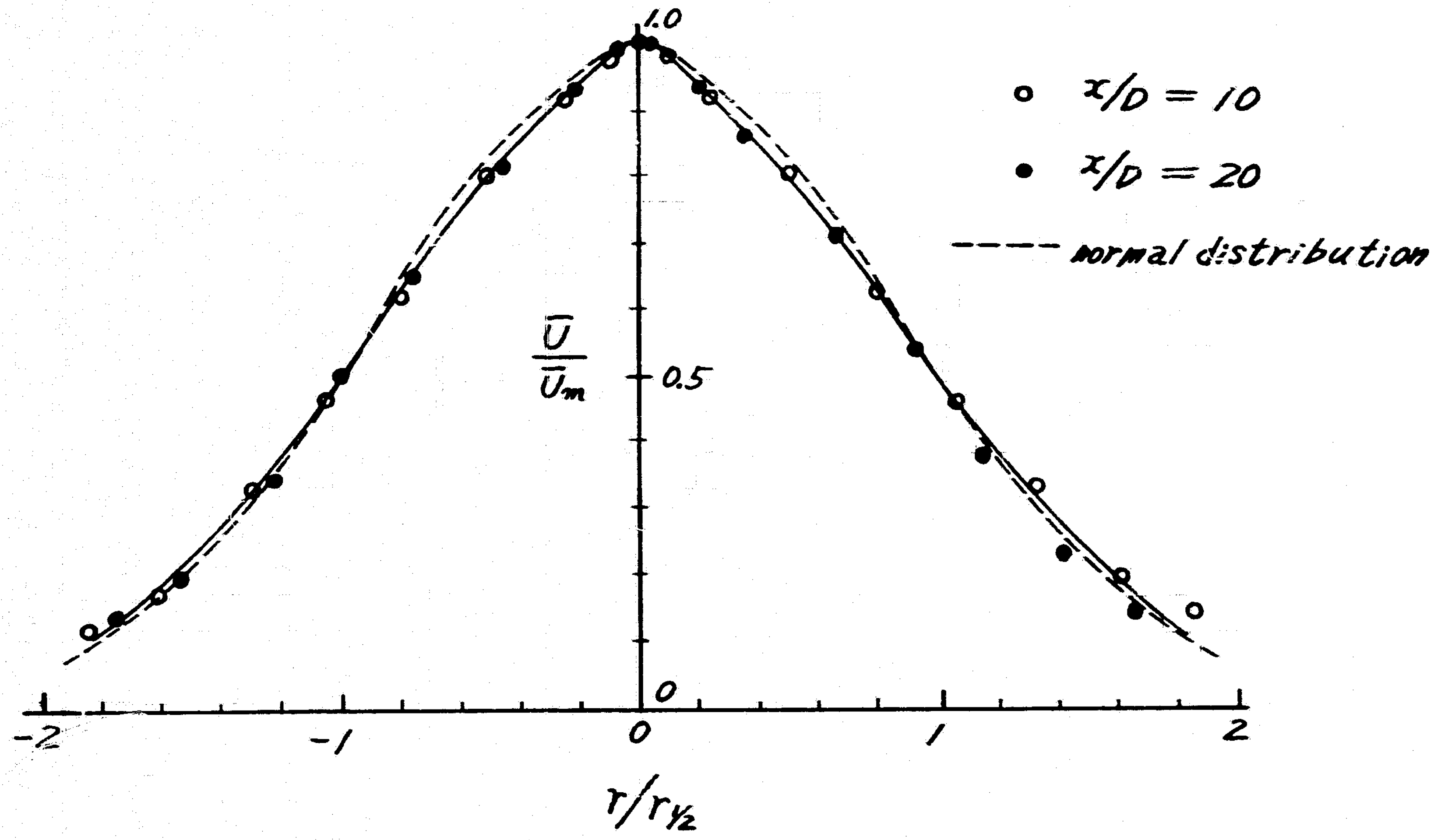
Sketch of a Jet Tunnel

Figure 9



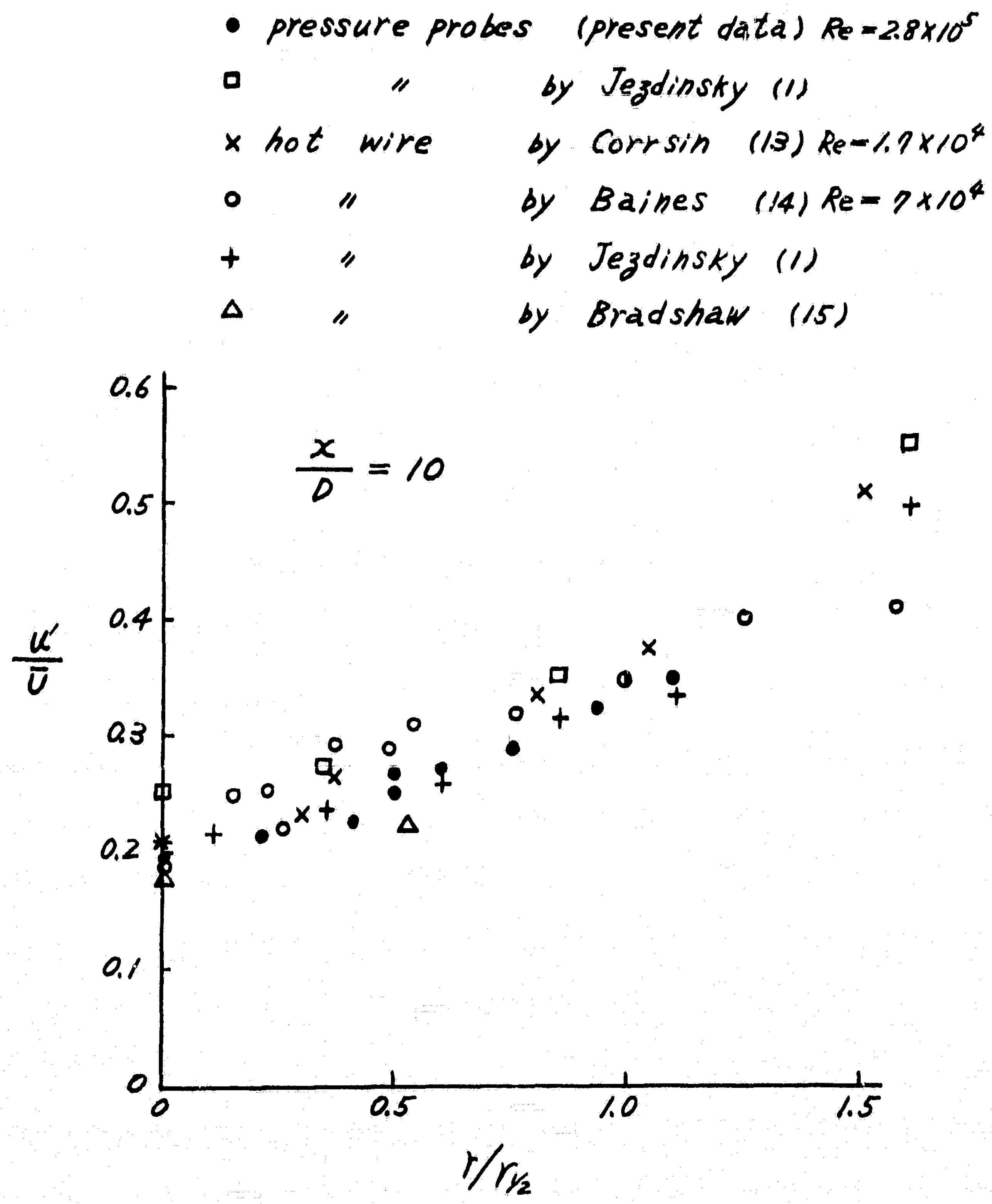
Longitudinal Velocity Distribution along Centerline of Jet

Figure 10



Mean Velocity Distribution across a Round Free Jet

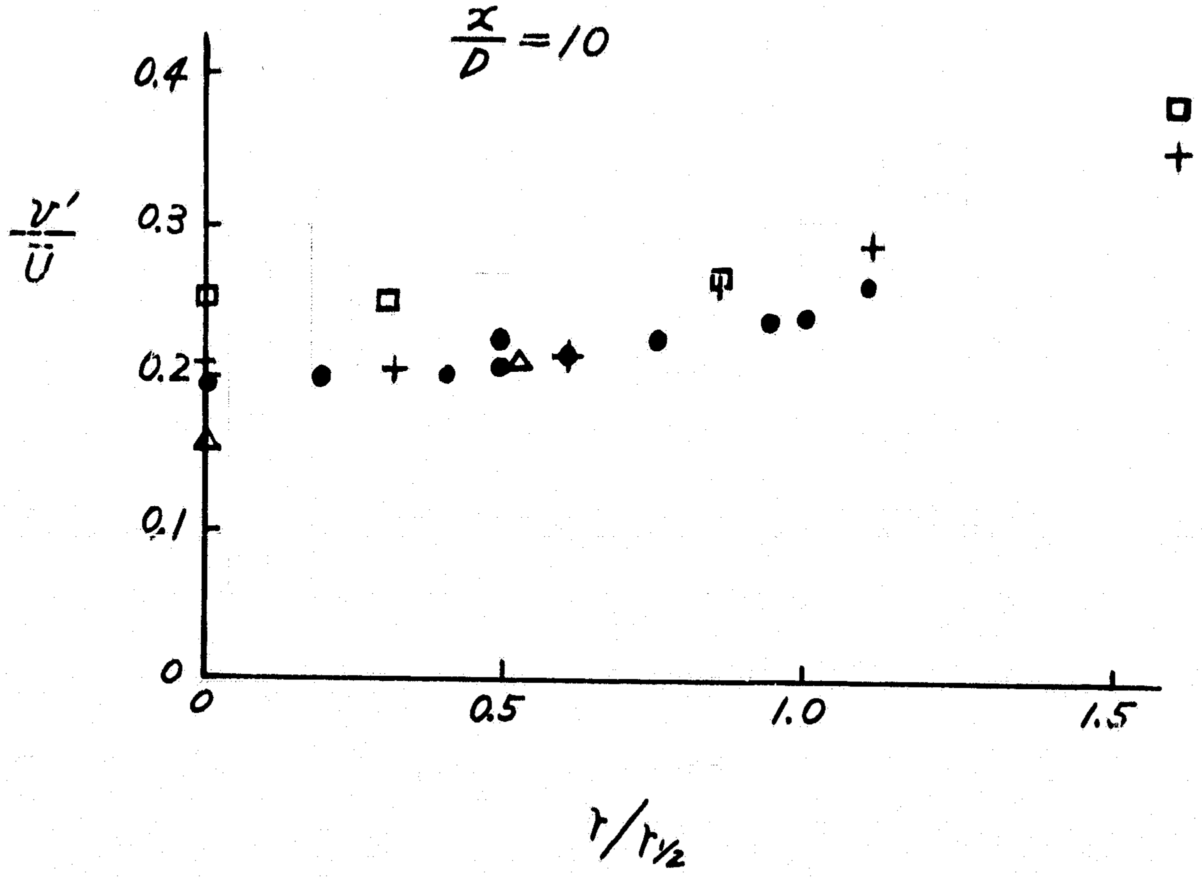
Figure 11



Distribution of Turbulent Intensity in a Jet

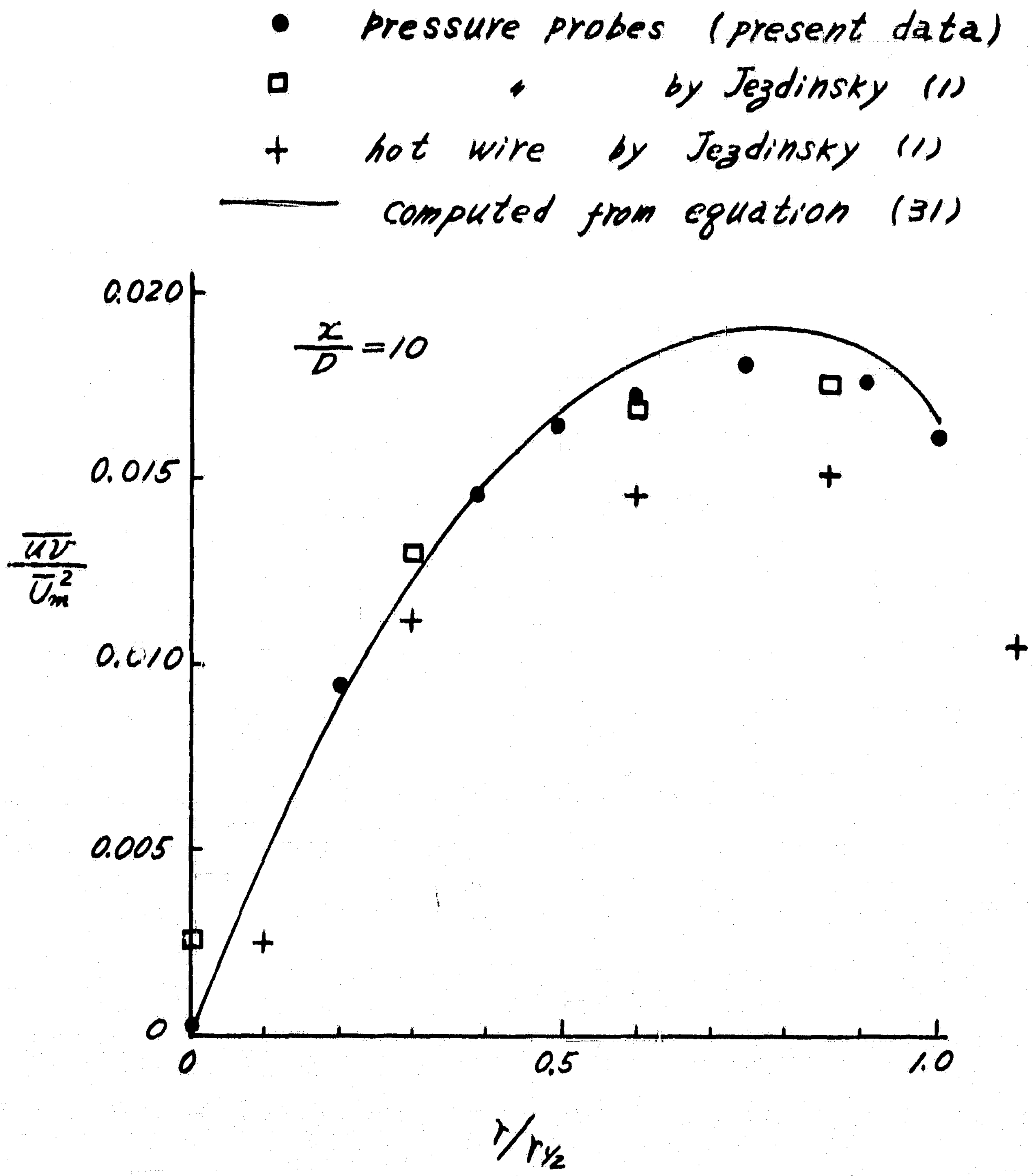
Figure 12

● Pressure Probes (present data)
□ 〃 by Jezdinsky (1)
+ hot wire by Jezdinsky (1)
△ 〃 by Bradshaw (15)



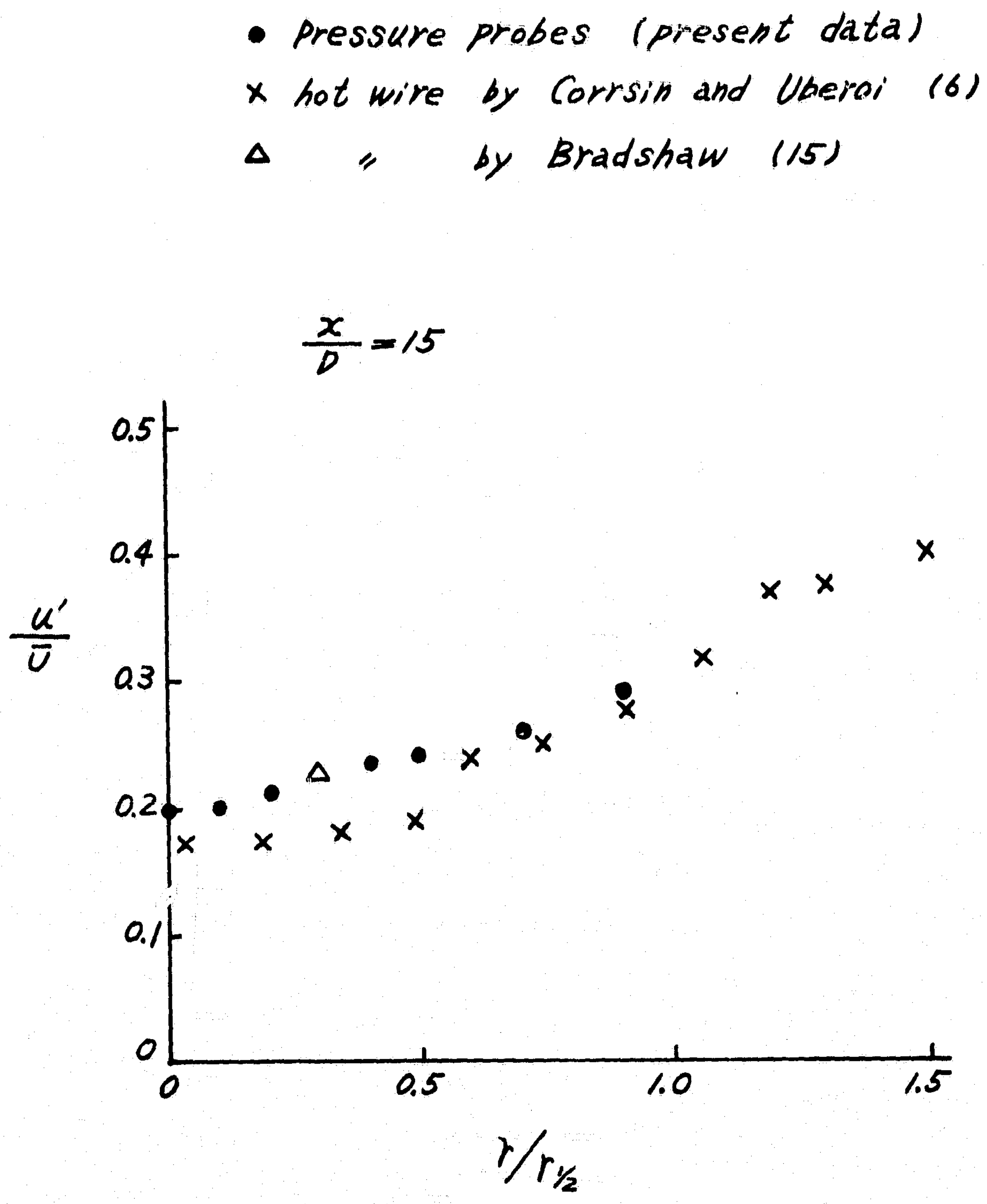
Distribution of Turbulent Intensity in a Jet

Figure 13



Turbulent Shear Stress Distribution in a Jet

Figure 14



Distribusion of Turbulent Intensity in a Jet

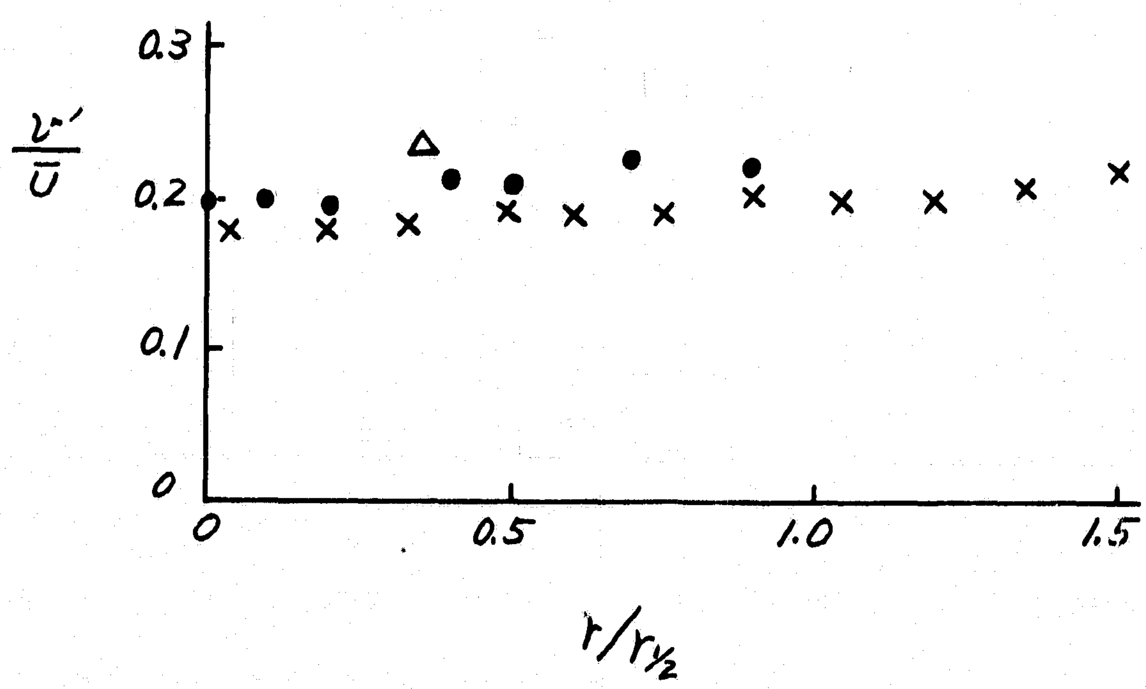
Figure 15

● pressure probes (present data)

× hot wire by Corrsin and Uberoi (6)

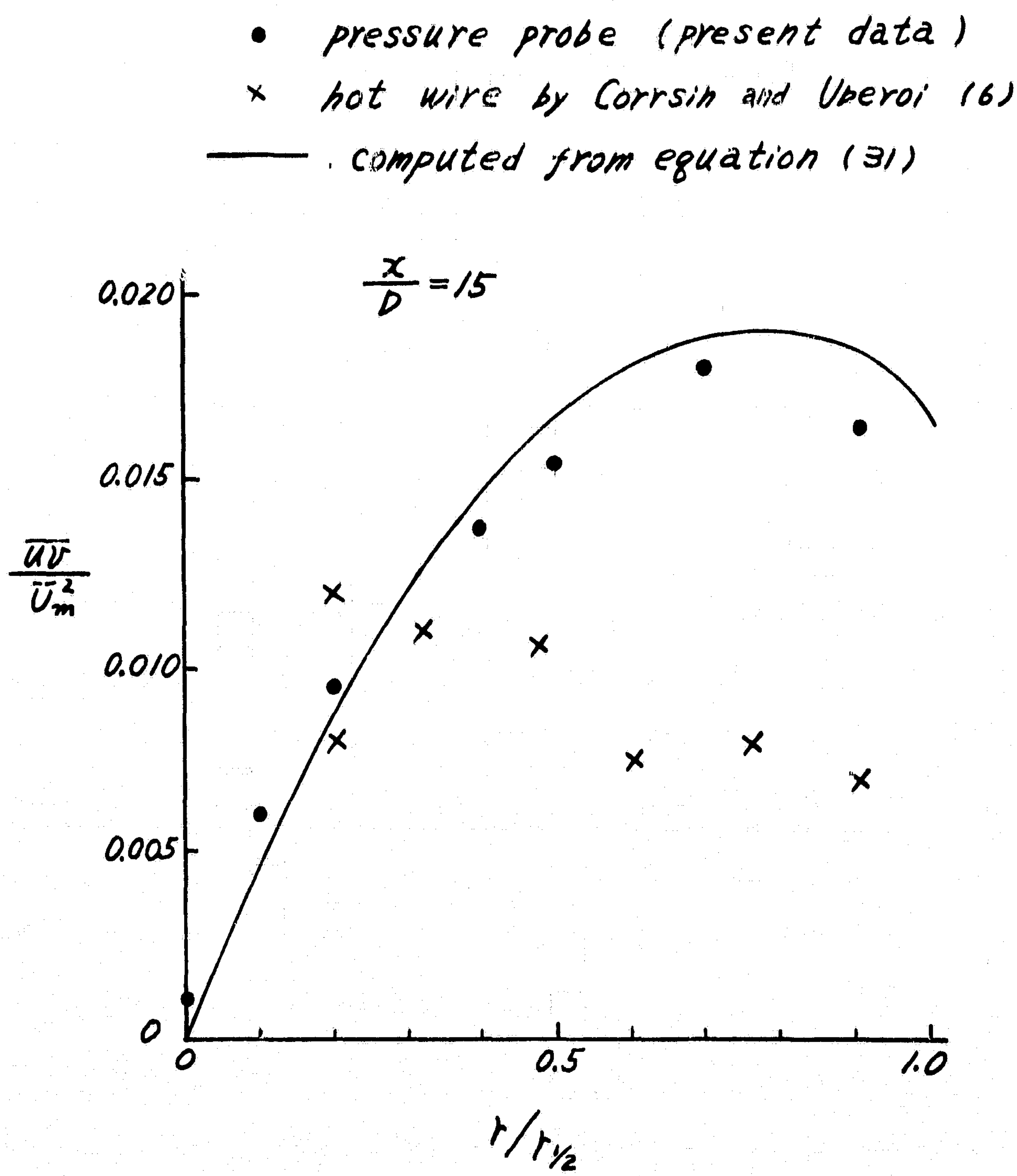
△ " by Bradshaw (15)

$\frac{x}{D} = 15$



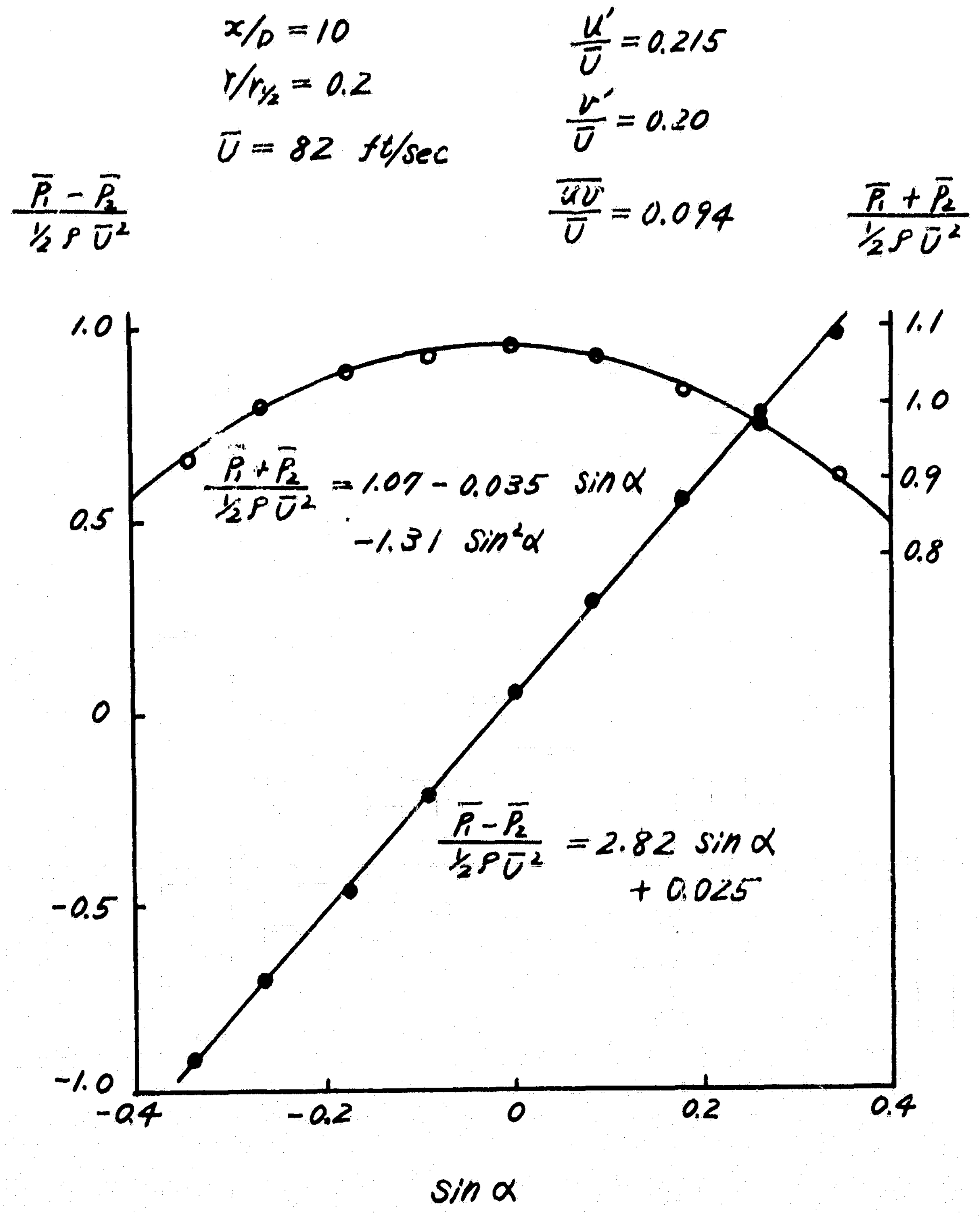
Distribution of Turbulent Intensity in a Jet

Figure 16



Turbulent Shear Stress Distribution in a Jet

Figure 17



Example of Pressure Probe Data

Figure 18